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Development of methods to model UAVS nonlinear automatic control systems

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ABSTRACT

When modeling Automatic Control Systems (ACS) of an unmanned aerial vehicle (UAV), it is often necessary to take into account the nonlinearity of an aircraft's reaction when the controls drift, as well as the strong influence of various destabilizing factors that make the system go out of linear mode. When known analytical and numerical methods are used to analyze dynamic systems, it is problematic to obtain general solutions that are valid for the variable parameters of the system under study and, at the same time, provide the required error value. A method has been developed to model dynamic processes in automatic non-linear UAV control systems based on linear approximation by parts and crosslinking of partial solutions with consideration of the initial conditions. An example of using the technique to model the transition characteristics of an ACS UAV with a single non-linear link is considered. Based on the analysis of errors in the calculation of the transition process, the effectiveness of the proposed approach is shown.

KEY WORDS: Unmanned aerial vehicle; UAV; non-linear automatic control system; linear approximation by parts; transitional process.

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Desarrollo de métodos para modelar sistemas de control automático no lineal de UAVS

RESUMEN

Al modelar sistemas de control automático (ACS) de un vehículo aéreo no tripulado (UAV), a menudo es necesario tener en cuenta la no linealidad de la reacción de una aeronave cuando los controles se desvían, así como la fuerte influencia de varios factores desestabilizadores que hacen que el sistema salga del modo lineal. Cuando se utilizan métodos analíticos y numéricos conocidos para analizar sistemas dinámicos, es problemático obtener soluciones generales que sean válidas para los parámetros variables del sistema en estudio y, al mismo tiempo, proporcionen el valor de error requerido. Se ha desarrollado un método para modelar procesos dinámicos en sistemas de control automático de UAV no lineales basado en aproximación lineal por partes y reticulación de soluciones parciales con consideración de las condiciones iniciales. Se considera un ejemplo de uso de la técnica para modelar las características de transición de un UAV ACS con un solo enlace no lineal. Con base en el análisis de errores en el cálculo del proceso de transición, se muestra la efectividad del enfoque propuesto.

PALABRAS CLAVE: Vehículo aéreo no tripulado; UAV; sistema de control automático no lineal; aproximación lineal por partes; proceso transitorio.

Introduction

To ensure flight safety and successfully solve a wide range of tasks based on unmanned aerial vehicles (UAVs), an effective UAV control system (ACS) is required at the physical level of the UAV communication network (Advanced Control of Aircraft, 2011; Austin, 2011). The development of such systems is impossible without an adequate mathematical model describing the operation of ACS in a wide range of parameters. A large number of studies consider the linear dynamics of UAVs (Lebedev and Chernobrovkin, 1973; Moiseev, 2013) and the linearity of their control systems (Zheng, et al., 2014; Kuriki and Namerikawa, 2014). The task of modeling and analyzing UAV ACS is significantly complicated when the non-linearity of the aircraft's response is manifested in case if the controls are deviated (Xiang et al., 2017; Raffo et al., 2010). In addition, the nonlinearity of processes in control systems is caused by the strong influence of various destabilizing factors that cause the system to exit the linear mode. In most cases, there are no convenient analytical expressions for describing nonlinear systems. Therefore, we have to use numerical methods for solving differential

equations that describe the dynamics of UAVs (Marino et al., 2016). This approach does not allow us to obtain general solutions for the variable parameters of the system under study. At the same time, the use of linearization of ACS characteristics based on the Taylor series is convenient for analytical research, but is often unacceptable because of the unacceptably high approximation error in a wide range of changes in impacts and response.

A promising approach is piecewise linear approximation of the system characteristics (Balashkov, 2010; Bogachev and Chapliga, 1976). The approach makes it possible to obtain analytical solutions for dynamic processes in control systems that are valid for individual linear sections of the characteristics of nonlinear parts of the system. Crosslinking of partial solutions is performed taking into account the initial conditions for each current section of the transition process up to the established mode.

The purpose of this work is to develop a method for modeling dynamic processes in nonlinear automatic control systems of UAVs based on piecewise linear approximation.

1. The dynamics of the UAV the longitudinal flight mode

The mathematical model of UAV movement in the longitudinal flight mode is described by a system of nonlinear differential equations (Gutsevich, 2018):

$$\begin{aligned} m \frac{dV}{dt} &= P \cos \alpha - X_a - mg \sin \theta, \\ mV \frac{d\theta}{dt} &= P \sin \alpha - Y_a - mg \cos \theta, \\ \vartheta &= \theta + \alpha, \\ I_z \frac{d\vartheta}{dt} &= M_z. \end{aligned}$$

In this system, m is the mass of the UAV, V – vector of linear velocity, P is the power of the engine thrust, α angle of attack, X_a is drag, Y_a is lifting force, g is the gravitational acceleration, ϑ – pitch, θ is the slope of the trajectory of the mass center of the aircraft, I_z is the moment of inertia of the UAV along the z -axis, M_z is the moment of the aerodynamic forces.

Similar nonlinear differential equations describe other UAV flight modes (start, landing, coordinated u-turn, spiral descent, etc.), while UAV dynamics in these modes

generally also have nonlinear properties. In general, the nonlinearity of UAV dynamics should be taken into account when modeling and developing UAV ACS.

2. Representation of the UAV ACS scheme based on the model of amplitude-phase signal conversion

The functional diagram of the aircraft height stabilization circuit is shown in Fig. 1 (Gordin, 2000).

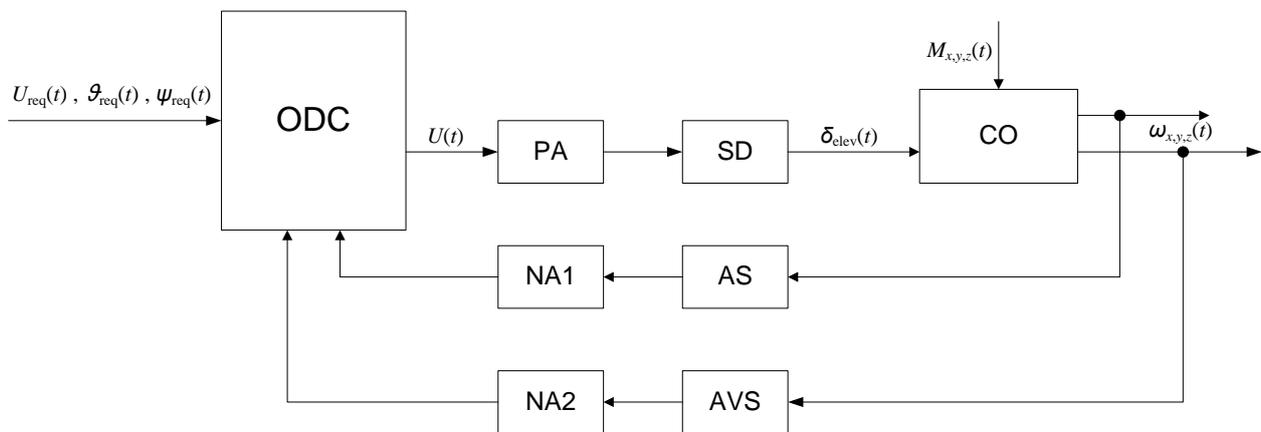


Figure 1. Functional diagram of the UAV control system

In Fig. 1 the following designations are accepted: ODC - onboard digital computer, PA - pre-amplifier, SD - servo drive, CO - control object; AS - angle sensor; AVS - angular velocity sensor; NA1, NA2 - normalizing amplifiers; $U_{req}(t)$, $\vartheta_{req}(t)$, $\psi_{req}(t)$ - the required values of roll, pitch, and yaw angles; $\delta_{elev}(t)$ - deviation angle of the elevator; $M_{x,y,z}(t)$ - disturbing effects; $U(t)$ - obtained pitch angle value; $\omega_{x,y,z}(t)$ - angular roll, pitch, and yaw speeds.

To analyze the two-channel scheme (Figure 1), we present each control channel based on the amplitude-phase signal conversion (APC) model (Vasilyev et al., 2011, 2013). The model of the converter with deviation control (Fig. 2) includes a similar APC* a control device (CD), a control path (CP), and a weight distributor (WD). The control device (CD) controls the amplitude and (or) phase of the input signal. The CP consists of a detector for deviation of the signal amplitude and (or) phase (D), as well as a filter (F). The values of the WD transmission coefficients determine the proportions of signal transmission from its

inputs to its outputs. The diagram shows: U_1, u_1 и U_2, u_2 – input (main, additional) and output signals, u – control signal.

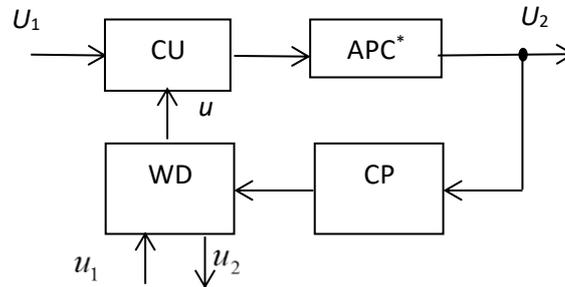


Fig. 2. Model of an amplitude-phase signal converter with deviation control

Transfer functions of APC model blocks (Fig. 2), equivalent to representing the control scheme (Fig. 1), have the form:

$$K_{APC^*} = K_{PA} \cdot K_{SD}, K_{CP}^{(1)} = K_{NA1} \cdot K_{AS}, K_{CP}^{(2)} = K_{NA2} \cdot K_{AVS}, K_{CU} = K_{WD} = 1.$$

3. UAV ACS modeling technique based on the APC model and piecewise linear approximation

To obtain analytical expressions of the dynamic characteristics (transients) of a non-linear UAV ACS, we approximate the nonlinear characteristic of the D based on piecewise linear functions (Kurilov et al. 2010, 2012). After approximation, the characteristic looks like the sum of M linear sections, where M is the maximum number of the approximation node. The equation of the approximating line segment for the node m : $f_{Dm} = K_m y + B_m$, where y is a parameter (amplitude or phase) of the output signal of the converter, K_m and B_m are approximation coefficients. Let's denote the transfer coefficient of K_{APC^*} as K^* and assume $u_1 = u_2 = 0$. The equation of the converter at the section m $[Y_m; Y_{m+1}]$

$$y_m = x_m - K^* [K_{ynp} M(p)(K_m y_m + B_m)],$$

where x is a parameter (amplitude or phase) of the input signal, $M(p)$ is a transmission coefficient of the CP filter, $p = d/dt$ is the operator, t is time, K_{CU} and K_{WD} are transmission coefficients of corresponding blocks.

We denote $\square N_m = K_y n_p K_m$ - the control coefficient and $B_m^* = K_y n_p B_m$. The Laplace image of the output parameter on the m section will take the form

$$y_m = x_m - K^* [M(p)(N_m y_m + B_m^*)]. \quad (1)$$

This is a linear differential equation of the APC at the interval $[m, m+1]$, presented in operator form. The general nonlinear equation of the converter can be represented by the sum of equations (1) for all parts of the approximation. Let's present the filter transfer function as

$$M(p) = A(p)/B(p) = \sum_{i=0}^l \alpha_i p^i / \sum_{i=0}^l \beta_i p^i, \quad (2)$$

where α_i, β_i are coefficients of the filter.

Substitute (2) in equation (1), which after the conversion will take the form

$$y_m \sum_{i=0}^l \beta_i p^i = x_m \sum_{i=0}^l \beta_i p^i - K^* \left[\sum_{i=0}^l \alpha_i p^i (N_m y_m + B_m^*) \right].$$

Let's replace the Laplace operator p with a differential d/dt in the resulting expression

$$\sum_{i=0}^l \beta_i \frac{d^i y_m}{dt^i} = \sum_{i=0}^l \beta_i \frac{d^i x_m}{dt^i} - K^* \left[\sum_{i=0}^l \alpha_i \frac{d^i (N_m y_m + B_m^*)}{dt^i} \right]. \quad (3)$$

Let's make an operator equation for (3) taking into account the initial conditions:

$$\begin{aligned} y_m(t) &\leftarrow Y_m(p), \quad y'_m(t) \leftarrow pY_m(p) - y_m(0), \quad y''_m(t) \leftarrow p^2 Y_m(p) - pY_m(0) - y'_m(0), \\ y^i_m(t) &\leftarrow p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0) - \dots \\ - y_m^{i-1}(0) &= p^i Y_m(p) - \sum_{k=1}^i y_m^{(k-1)}(0) p^{i-k}. \end{aligned}$$

We assume that all derivatives except the first one are null: $y^{(i)}_m(0) = 0, i > 1$. Similarly, we present an image of the impact and its derivatives. Then

$$x^{(i)}_m(t) \leftarrow p^i X_m(p) - p^{i-1} x_m(0) - p^{i-2} x'_m(0). \quad (4)$$

Given (4), the expression (3) will take the form

$$\begin{aligned} &\sum_{i=0}^l \beta_i [p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0)] = \\ &= \sum_{i=0}^l \beta_i [p^i X_m(p) - p^{i-1} x_m(0) - p^{i-2} x'_m(0)] - \\ &- K^* N_m \sum_{i=0}^l \alpha_i [p^i Y_m(p) - p^{i-1} y_m(0) - p^{i-2} y'_m(0)] - \\ &- \frac{K^* B_m^*}{p} \sum_{i=0}^l \alpha_i p^i. \end{aligned}$$

From the resulting equation, we express $Y_m(p)$:

$$Y_m(p) = \frac{pB(p)X(p) - K^* B_m^* A(p) + pC_m(p)}{p[B(p) + K^* N_m A(p)]}, \quad (5)$$

$C_m(p) = C^{(y)}(p) - C^{(x)}(p) - C^{(G)}(p)$ - polynomial of initial conditions.

$C^{(y)}(p) = [\tilde{B}_1(p) + K^* N_m A_1(p)]y_m(0) + [\tilde{B}_2(p) + K^* N_m A_2(p)]y_m'(0)$ - polynomial of initial response conditions,

$C^{(x)}(p) = \tilde{B}_1(p)x(0) + \tilde{B}_2(p)x'(0)$ - polynomial of initial impact conditions,

$$C^{(G)}(p) = \begin{cases} K^* B_m^* A_1(p), m \neq m_n \\ 0, m = m_{ini}, \end{cases} \quad \tilde{B}_k(p) = \sum_{i=k}^I \beta_i p^{i-k}, \quad A_k(p) = \sum_{i=k}^I \alpha_i p^{i-k}.$$

Initial conditions for different m take the following values:

1) $m = m_n$, then $x_{m_n}(0) = y_{m_n}(0) = 0$, $x'_{m_n}(0) = y'_{m_n}(0) = 0$.

For APC with filters of the second and higher orders ($I \geq 2$) $y'_{m_n}(0) = 0$, it corresponds to zero initial values of the stresses on the reactive elements of the filter (Lebedev and Chernobrovkin, 1973).

2) For each subsequent section ($m \neq m_{ini}$), the APC response values at the border of the sections are the same:

$y_m(0) = Y_{m+\bar{q}_m}$, $y'_m(0) = y'_{m-1+2\bar{q}_m}(t_{m-1+2\bar{q}_m})$, where t_m is the end time of a particular solution, $\bar{q}_m = q[y_m(t-\Delta) - y_m(t)]$ - the direction of the transition process ($\bar{q}_m = 1$ when decreasing and $\bar{q}_m = 0$ when increasing), $\Delta \rightarrow 0$, $q(y)$ - the CPLF of inclusion, equal to 1 at $y \geq 0$ and 0 at $y < 0$, $y'_m(t) = \frac{y_m(t+\Delta) - y_m(t)}{\Delta}$. To calculate t_m , we must determine the abscissa of the intersection points $y_m(t)$ with Y_m line.

The initial value of the impact and its derivative on the m -th section are determined by the time shift: \tilde{t}_m : $x_m(0) = x(\tilde{t}_m)$, $x'_m(0) = x'(\tilde{t}_m)$. For each current section, the shift will consist of the sum of the transition durations for each of the previous sections. Let's denote $k=0..K-1$ is the number of a particular transient solution, K is the total number of particular solutions ($k=0$ corresponds to $m_k=m_0=m_{ini}$, $k=K-1$ corresponds to $m_{K-1}=m_{end}$). Thus, the time shift of a particular transient solution is defined as

$$\tilde{t}_{m_k} = \sum_{\tilde{k}=0}^{k-1} \tilde{t}_{\tilde{k}}, \quad (6)$$

$$\tilde{t}_{m_n} = 0.$$

4. UAV ACS modeling based on the APC model and piecewise linear approximation

Based on the developed method, the simulation of the transition characteristics of the UAV ACS was performed. Accepted $K_y=1$, $n_p=2$, $K^*=1$. The characteristic of the filter F, which describes the inertia of the feedback circuit of the control loop of the system, is given by the expression $M(p) = (1 + p)^{-1}$. The characteristic of the detector D, describing the non-linearity of the ACS, is approximated by two and three straight lines (Fig. 3a and 3b). The calculated transient characteristic of the UAV ACS when approximating the D with two and three linear sections is shown in Fig. 4. A solid line highlights the component formed by the initial section of the characteristic D and described by the expression $y_1(t) = e^{-2t}$. The dotted line marks the next section of the transition characteristic described by the expression $y_0(t - t_1) = 0,125 + 0,375e^{-4t}$, where $t_1 = 0.34$ s is the time shift of the second partial solution. The result of calculating the transition process for three parts of the D approximation is shown by a dashed line (time shifts of the second and third partial solutions ($t_2 = 0.087$; $t_1 = 0.43$)). The figure shows that the error in the representation of D by three sections is significantly less, especially at the end setting stage.

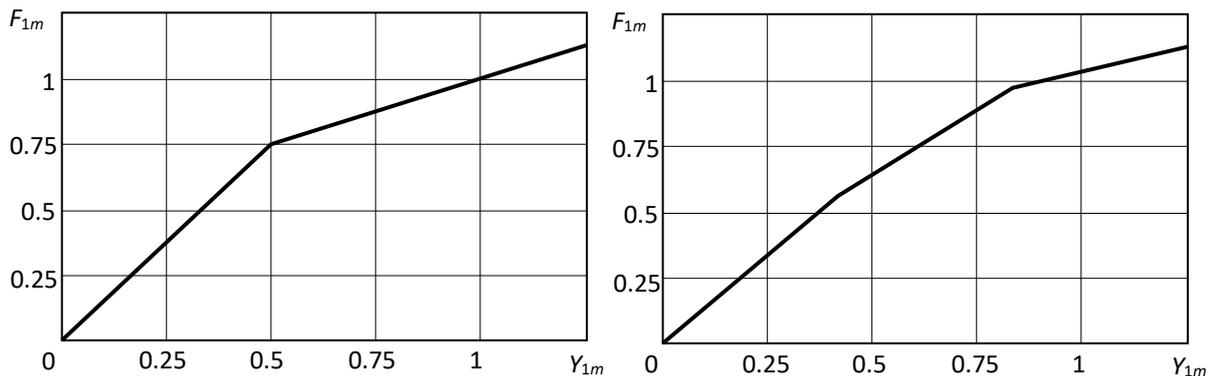


Fig. 3. Approximation of the characteristics of a nonlinear UAV ACS link: a) two linear sections; b) three linear sections

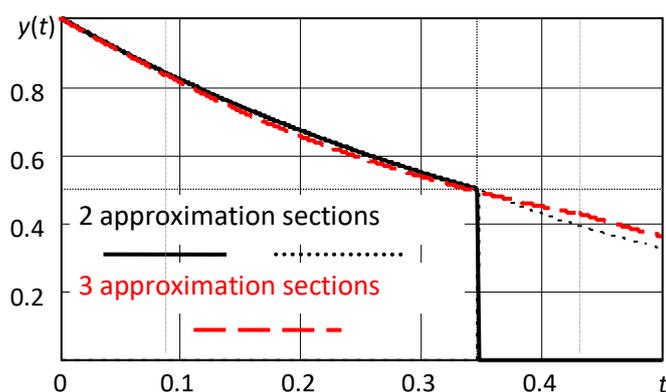


Fig. 4. Transient characteristics of a UAV ACS when approximating a nonlinear link with two or three linear sections

Conclusions

The task of modeling and analyzing UAV ACS is significantly complicated by factors such as the nonlinearity of the aircraft's response when the controls are deviated, as well as the strong influence of various destabilizing factors that cause the system to exit the linear mode. Known numerical and analytical methods do not allow us to obtain general solutions for the variable parameters of the system under study and at the same time provide the required error value.

A method for modeling dynamic processes in nonlinear UAV automatic control systems based on piecewise linear approximation is developed. The approach makes it possible to obtain analytical solutions for dynamic processes in control systems that are valid for individual linear sections of the characteristics of nonlinear parts of the system. Crosslinking of partial solutions is performed taking into account the initial conditions for each current section of the transition process up to the established mode. In the framework of the proposed approach, a representation of the ACS scheme that performs the function of UAV height stabilization is performed on the basis of a model of amplitude-phase signal conversion with deviation control. Analytical expressions describing dynamic processes in UAV ACS with a single nonlinear link are obtained based on the signal conversion model and piecewise linear approximation.

Based on the developed method, the UAV ACS with a nonlinear link represented by two and three linear sections is simulated. It is found that the error in calculating the

transition process when presenting the link characteristic by three approximation sections is significantly less, especially at the end setting stage. Thus, the effectiveness of the proposed approach is shown. When analyzing the dynamic modes of a specific UAV control system, approximated by the signal conversion model with deviation control, its characteristics can be obtained by substituting the corresponding numerical coefficients of the device into general expressions of the dynamic characteristics of the converter.

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