

Forecasting of consumer demand with the use of multi-factor dynamic models

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Abstract

In this paper, it is proposed to use a multifactor nonlinear dynamic model to predict consumer demand. For this purpose, a method of varying differences is proposed in order to reduce the autocorrelation of time series. According to the multifactor dynamic model of consumer demand, the authors have proposed a model for forecasting the demand for goods depending on the given values of the total turnover, price indices and the influence of other factors on the formation of consumer demand of the population. The character of dependencies and nonlinear trends was studied, what has allowed qualitative models to obtain.

Keywords: Dynamic Models, Correlation, Time Series.

Pronóstico de la demanda del consumidor con el uso de modelos dinámicos multifactoriales

Resumen

En este documento, se propone utilizar un modelo dinámico no lineal multifactorial para predecir la demanda de los consumidores. Para este propósito, se propone un método de diferencias variables con el fin de reducir la autocorrelación de las series de tiempo. De acuerdo con el modelo dinámico multifactorial de la demanda del consumidor, los autores han propuesto un modelo para pronosticar la demanda de bienes en función de los valores dados de la facturación total, los índices de precios y la influencia de otros factores en la formación de la demanda de consumo de la población. Se estudió el carácter de las dependencias y las tendencias no lineales, lo que ha permitido obtener modelos cualitativos.

Palabras clave: modelos dinámicos, correlación, series de tiempo.

1. Introduction

With the establishment of market relations, the role of consumer demand in the market for goods and services is changing. Demand can be defined as the desire and ability of a consumer to buy a product or service at a specific time and place. The definition of demand includes an assessment of the quantity and structure of the current demand and the prospect of its change. The study of demand provides information to predict the capacity and structure of the market, as well as the expected changes in improving the geography of production and consumption of goods and services. Demand forecasting is the determination of the possible future demand for goods and services in order to better adapt business entities to the current market conditions. The demand forecast is a theoretically substantiated dynamic multi-factor model about a still unknown volume and structure of demand. Forecasting associates past experience with the volume and structure of demand with predicting their future state (Konoreva and Guseva, 2009; Stolyarova, 2008).

2. Discussion

A dynamic multifactorial model of consumer demand is usually expressed by a multiple regression equation and is based on time-dependent data on the sale of goods. It is known that consumer demand is formed in dynamics under the influence of a large number of factors. Therefore, a lot of independent variables need to be included in the dynamic model. However, a significant increase in the number of factors in the dynamic model will require an increase in the initial information. And this is practically impossible under the conditions of a dynamic analysis of demand. That is why, in the figurative expression of the American mathematician R. Bellman, when building a multifactor dynamic model, there are two dangers before the researcher: over-complication bog and over-simplification trap. The danger of over-complication is due to the presence of many factors that influence consumer demand. The limited information expressed in the absence of large time series pushes the researcher into a trap of over-simplification, to a significant reduction in the number of factors in a dynamic model of consumer demand. Building a multi-factor model to study consumer demand is not an easy task. We offer to solve it in four stages (Potapova and Minyova, 2004).

1. Factor analysis of consumer demand.
2. Application of methods for measuring the impact of factors influencing consumer demand and taken into account.
3. Application of methods for measuring the influence of other factors on consumer demand.
4. Compilation of a dynamic multi-factor demand model.

The analysis of dynamic factors will be considered in two aspects: the classification of factors and the methods of aggregation of factors (Mandruck and Kadochnikova, 2014).

2.1. Classification of factors

Consumer demand changes in dynamics under the influence of a large number of factors. Concretization of these factors and giving them a quantitative characteristic are necessary for creating a dynamic multi-factor model for studying demand. We list the main groups of factors that determine the change in consumer demand over time:

- a) The volume and structure of family income, the change in these indicators over time;

- b) The saturation of the family budget with goods, mainly durable goods;
- c) The composition and size of the family, the change in these indicators over time;
- d) Price fluctuations for consumer goods and services;
- e) Changes in the volume and structure of the public consumption fund;
- f) Saturation of the market with goods, the quality of goods, the state of the trading network, the culture of customer service. To characterize the saturation of the market with goods there can be used such indicators as the production of personal goods, turnover in terms of per capita, etc.;
- g) Migration of the population from a region to another region, from the countryside to the city;
- h) A change in fashion, consumer habits, caused by new working or life conditions, material and cultural level of the population. The problem of studying consumer habits, their changes over time and giving this factor quantitative certainty is the most difficult moment in building a dynamic multi-factor model. The complexity of this problem lies in the fact that consumer habits cannot be expressed by any quantitative indicator (Zatonski and Sirotina, 2014).

The above list of factors of a dynamic model for studying consumer demand is certainly incomplete. However, in this form, it helps to clarify the essence of the studied dynamic patterns of consumer demand. The information base of this study is the official statistics of the Russian Federation for 1991-2016. Due to the limited official statistics, both in sample composition and in size, the number of independent variables is reduced to two, to ensure the statistical reliability of the regression analysis results. Available are statistics on such demand factors as total public consumption and data on price changes for basic food products (Polovkina and Grigoreva, 2017).

2.2. Aggregation of factors

It is clear that all of these factors cannot be included in the dynamic model of demand. It is also wrong to exclude some factors from the study. For example, the demand for potatoes depends not only on the price of potatoes, but also on the prices of all other food products, as well as industrial goods and services. The inclusion of the prices of all goods and services in a dynamic model of demand for potatoes is practically impossible; at the same time, the method of exclusion from this model of prices for certain types of goods and services is also erroneous. This problem can be solved by replacing individual price indices with aggregate indices. It is known that the general price index can replace several dozen individual indices. As a result of aggregation, the number of factors is sharply reduced. However, the problem of accounting for all factors can be solved only if we manage to divide all factors into two groups (recorded and others) and quantify their influence on the formation of consumer demand. Factors taken into account usually include those with which we can control consumer demand, such as income and prices. Other factors are components that cannot be individually measured (fashion, consumer habits, etc.) (Gazizov, 2016).

3. Methodology

We study the change in consumer demand for shoes under the influence of such factors as total consumption and price changes. To solve this problem, data for the 16-year period was used, based on the statistical compilations Regions of Russia. Socio-economic indicators:

y - The sale of footwear per capita at current prices;

x_1 - Total turnover in current prices (per capita);

x_2 - The product of chain price indices of shoes, since 1991;

x_3 - The product of common chain price indices since 1991.

The product of sequential chain indices allows the values of the basic indices to obtain. We take 1991 as the base year (Ismagilov and Khasanova, 2017). To measure the impact of total consumption and consumer prices, the following indicators were calculated for this information (by year):

a) The sale of shoes in constant prices of 1991:

$$\frac{y}{x_2} = u$$

b) The total turnover in constant prices of 1991:

$$\frac{x_1}{x_3} = v_1$$

c) Actual price index:

$$\frac{x_2}{x_3} = v_2$$

The task is reduced to the definition of consumer demand at constant prices depending on the total turnover in constant prices and the actual price index. It should be noted that the traditional methods of determining correlation are designed exclusively for random samples, and therefore they are not applicable in dynamics, where the correlation between neighboring members of the time series (autocorrelation) is clearly expressed. In order to reduce the autocorrelation of time series, a varying differences method is proposed. According to this method, the original data is replaced by the first differences. As a result, a linear trend is excluded from the time series (Kadochnikova et al., 2017). If initial data are expressed in logarithms, then using the varying differences method, the tendency in the form of an exponential curve is excluded from the time series. The demand function is inherently non-linear, its parameters characterize elasticity, therefore logarithms ($\lg u, \lg v_1, \lg v_2$) should be taken for the initial data (u, v_1, v_2), and to reduce autocorrelation, their first differences should be determined:

$$\lg u_{(i)} - \lg u_{(i-1)} = z_{(i)}, \quad \lg v_{1(i)} - \lg v_{1(i-1)} = l_{1(i)}, \quad \lg v_{2(i)} - \lg v_{2(i-1)} = l_{2(i)}$$

As a result of using the decimal logarithm, the autocorrelation factors of shoe sales volumes decreased from 0.9-0.98 (for u, v_1, v_2) to 0.3-0.4 (for z, l_1, l_2). The desire to reduce the autocorrelation coefficient would be explained further by the fact that the influence of other factors unaccounted for in the model will be evaluated in the future when modeling the trend, which will serve as a generalizing indicator of significant unrecorded factors. Elimination of the general trend allows us to reveal the true influence of the factors chosen by us (total consumption and price index) on the change in demand for goods (Bezruchko and Smirnov, 2006).

The resulting index values (z_i) are changed under the influence of both recorded factors (l_{1i} and l_{2i}) and other factors (fashion, consumer habits, production, economic policy, historical features of the development of the country, etc.). It is natural to assume that the group of other factors has a decisive influence on the formation of the general trend of sales indices [$z(t)$]. The turnover and the

real price index are also involved in formation of development trends $z(t)$ in the form of a general trend $[l_1(t) \text{ and } l_2(t)]$.

Chow's test for the presence of structural changes showed with a reliability of more than 95% that during the period under review chain indices $[z, l_{1(i)}, l_{2(i)}]$ have two different non-linear development trends. For this reason, the general trends of chain indices were calculated using a second-order parabola in two parts of the period studied (I - 1991–2006 and II - 2007–2016).

$$z(t)_I = 0,14704 - 0,01882 t - 0,000003 t^2$$

$$z(t)_{II} = 0,05982 - 0,00711 t + 0,00021 t^2$$

$$l_1(t)_I = 0,09853 - 0,00967 t - 0,000003 t^2$$

$$l_1(t)_{II} = 0,04302 - 0,00606 t + 0,00033 t^2$$

$$l_2(t)_I = 0,03869 - 0,00683 t - 0,000002 t^2$$

$$l_2(t)_{II} = -0,00837 + 0,00411 t - 0,00039 t^2$$

All models and their factors are statistically significant according to F-test and t- test, respectively, with a reliability of more than 95%. The trends calculated $[z(t), l_1(t), l_2(t)]$ break up zigzag waves of indices $(z_i, l_{1(i)}, l_{2(i)})$ into two parts.

As a result of the calculation of general trends in development $z_i, l_{1(i)}, l_{2(i)}$, deviations are calculated:

$$z_{(i)} - z(t_i) = m_{(i)}, \quad l_{1(i)} - l_1(t_i) = s_{1(i)}, \quad l_{2(i)} - l_2(t_i) = s_{2(i)}$$

Using the values $m_{(i)}, s_{1(i)}, s_{2(i)}$ the impact of the recorded factors (turnover and the real price index) on consumer demand is determined:

$$m(s_1 s_2) = a_1 s_1 + a_2 s_2 \quad (2.0)$$

Function (2.0) is static by its character. Calculations of its parameters are made by the method of least squares. As a result of the calculations, the following equation for the demand for shoes was obtained:

$$m(s_1 s_2) = 1,1010914 s_1 - 1,0201386 s_2 \quad (2.1)$$

The parameters of this equation are the net elasticity factors of the total turnover (a_1) and of the price index (a_2) . They show that with an increase in total turnover (at constant prices) of 1%, the cost of buying shoes (at constant prices) increases by 1.10%, and with an increase in the real price index of 1%, the cost of purchasing shoes (at constant prices) decrease by 1.02%. As an example, we give the net elasticity factors for demand from the total turnover (a_1) and from the price (a_2) calculated with the use of the time series of trade for a 16-year period (Medvedeva et al., 2017).

Goods	(a_1)	(a_2)
Fish	0.61	-0.72
Cheese	1.18	-1,18
Canned fish	1.27	-1.47
Canned vegetables	1.32	-1,26
Confectionery	1.02	-1.37
Tea	0.21	-0.36

Groats and legumes	0.43	-0.11
Macaroni products	0.43	-0.34
Potatoes	0.44	-0.35
Vodka	0.88	-0.87
Wine	1.54	-1.40
Beer	0.76	-0.39
Manufactured goods	1.53	-1.63
Hat	1.14	-1.38
Knitwear	1.46	-1.41
Leather, textile shoes	1.10	-1.02
Rubber shoes	0.37	-0.37
Perfumery	0.91	-0.61
Haberdashery	1.08	-0.88
Tobacco goods	0.55	-0.52
Matches	0.49	-0.48
Furniture	1.86	-1.34
Sport goods	0.61	-0.60
Electric appliances	1.34	-1.31
Home appliances	1.47	-1.29
Toys	1.16	-0.71

Table 1 - Net cost elasticity factors with regard to total turnover and price

The calculations showed that the net elasticity factors of purchases from the general goods turnover are positive, and the net elasticity factors of expenses from the price are negative (Table 1). At the same time, such goods as cereals, pasta, potatoes and tea have the least sensitivity to price increase, and the greatest sensitivity is inherent to canned goods, knitwear, wine and confectionery. However, demand for the same basic products grows more slowly than the growth of total turnover, as the elasticities were less than 1. The findings are fully consistent with the theory of consumption and empirically confirm the economic laws. The above is a method for studying the effect on consumer demand of such recorded factors as general turnover and the price index. These factors are decisive only in the formation of deviations from the time series of sales from the general development trend. But how to measure the influence of such factors as production, economic policy in the country, fashion and consumer habits on the formation of a time series for sales of goods? This question must be answered in this section (Ismagilov and Khasanova, 2015).

The function (2.0) is written in the notation z, l_1 and l_2 :

$$z - z(t) = a_1[l_1 - l_1(t)] + a_2[l_2 - l_2(t)] \quad (3.0)$$

It should be noted that the time trends in this equality (3.0) are calculated by the equation of a straight line for the second period.

We open brackets in equality (3.0) and move $z(t)$ to the right side:

$$z = z(t) - a_1 l_1(t) - a_2 l_2(t) + a_1 l_1 + a_2 l_2 \quad (3.1)$$

We add and subtract $\overline{z(t)}$, $a_1 \overline{l_1(t)}$ and $a_2 \overline{l_2(t)}$ in equation (3.1):

$$\begin{aligned}
z &= \left\{ \overline{z(t)} + \left[z(t) - \overline{z(t)} \right] \right\} + \\
&+ \left\{ a_1 \overline{l_1(t)} + \left[-a_1 l_1(t) - a_1 \overline{l_1(t)} \right] \right\} + \\
&+ \left\{ a_2 \overline{l_2(t)} + \left[-a_2 l_2(t) - a_2 \overline{l_2(t)} \right] \right\} + a_1 l_1 + a_2 l_2.
\end{aligned} \tag{3.2}$$

We introduce the notation:

$$\overline{z(t)} - a_1 \overline{l_1(t)} - a_2 \overline{l_2(t)} = a_0. \tag{3.3}$$

Because $\overline{z(t)} = \bar{z}$, and

$$\overline{z(l_1 l_2)} = a_1 \overline{l_1(t)} + a_2 \overline{l_2(t)},$$

Then equality (3.3) can be written:

$$a_0 = \bar{z} - \overline{z(l_1 l_2)} \tag{3.4}$$

or

$$a_0 = \overline{z(t)} - \overline{z(l_1 l_2)}. \tag{3.4a}$$

Therefore, the parameter a_0 characterizes the change in the chain of sales indices under the influence of other factors on average for the entire study period.

We write function (3.2) in the following manner:

$$\begin{aligned}
z &= a_0 + \left\{ \left[z(t) - \overline{z(t)} \right] + \left[-a_1 l_1(t) + \overline{a_1 l_1(t)} \right] + \right. \\
&\left. + \left[-a_2 l_2(t) + \overline{a_2 l_2(t)} \right] \right\} + a_1 l_1 + a_2 l_2.
\end{aligned} \tag{3.5}$$

Function 3.5 will be divided into two equations:

$$\hat{z} = a_0 + a_1 l_1 + a_2 l_2 \tag{3.6}$$

$$\begin{aligned}
\check{z} &= \left[z(t) - \overline{z(t)} \right] + \left[-a_1 l_1(t) + \overline{a_1 l_1(t)} \right] + \\
&+ \left[-a_2 l_2(t) + \overline{a_2 l_2(t)} \right]
\end{aligned} \tag{3.7}$$

Function (3.7) is obtained by subtracting of equation (3.6) from (3.5) by years:

$$z_i - \hat{z}_i = \check{z}_i.$$

Values \check{z}_i show changes in chained sales indices under the influence of other factors in deviations from the average value a_0 . These values vary linearly with time and are equal to zero in the middle of the period. $(t - \bar{t})$. Function values \check{z} (3.7) can be calculated by the formula:

$$\check{z} = a_3 (t - \bar{t}), \tag{3.8}$$

Where is the time trend parameter $a_3 = \check{z}_i - \check{z}_{i-1}$.

4. Results

The dynamic multi-factor consumer demand model (3.1) is written by the equation:

$$z = a_0 + a_1 l_1 + a_2 l_2 + a_3 (t - \bar{t}) \quad (4.0)$$

To determine consumer demand for the planned period, it is necessary first of all to calculate the indices of sales of goods under the influence of taken into account and other factors according to a function of type (4.0). Prediction of prices and their indices depends on the type of tasks and can be carried out using various qualitative and quantitative methods. This equation includes parameters a_1 and a_2 calculated for the entire base period; time trend parameter (a_3) calculated for the second part of the base period; and free member (a_0) should be determined for the planning period (Grigalashvili et al., 2016). The need to calculate the parameter a_0 for the planning period is due to the fact that its value is mainly determined by the size z . This follows from the formula for determining the free term (3.4a). In order to obtain the most accurate calculation of a_0 , it is necessary to use interpolation methods to determine $\overline{z(t)}$ for the planning period. In the equality (3.4a), the value $\overline{z(t)}$ should correspond to the middle of the analyzed period of time. If for calculations for the prospective period for the middle we take the last year of the reporting period, then the beginning of the period is accordingly postponed. For this period, $\overline{z(t)}$ is equal to $z(t_n)$ calculated for the last year of the reporting period (n). It should be noted that since the general trend of change in chained sales indices in the planning period is not linear, the values $z(t)$ should be aligned with a curvilinear function, for example of the type:

$$z(t) = a + b \lg t$$

We take the calculated value $z(t_n)$ for the last year of the reporting period as $\overline{z(t)}$ for the whole period (basic and prospective); the values $\overline{l_1}$ and $\overline{l_2}$ written in functions (3.4a) should be calculated for the entire period (basic and prospective). For the planned period, chained indices of sales of goods should be calculated using equation (4.0); $\overline{l_1}$ and $\overline{l_2}$ are determined for the same period, the middle of the study period is shifted to the last year of the reporting period. The transition from chained sales indices to absolute values is made according to the formula:

$$\lg u_t = \lg u_{t_i} + \overline{z_{t_i}} (a_0 l_1 l_2) t + a_3 t, \quad (4.1)$$

Where t is the serial number of the year, starting from the last year of the reporting period (t_i).

5. Conclusions

This paper presented the results of consumer demand modeling using the example of the indicator of shoe sales in Russia for the period of 1991-2016. The character of dependencies and nonlinear trends, which allowed qualitative models to obtain, were studied. The first-difference method was

also used, which made it possible to reduce autocorrelation and increase the reliability of statistical estimates. Based on the calculations of the model parameters for different groups of goods, the elasticities of demand for them were calculated with regard to such factors as the change in the total turnover and the price index. The findings confirm the economic theory of consumption of food products by the population, according to which the growth of general consumption signals on an increase in the welfare of society, when the share of consumption of basic foodstuffs (potatoes, cereals, tea) in total expenditures decreases, but the demand for such products remains insensitive to price changes. According to the multifactor dynamic consumer demand model (4.0), it is possible to make calculations for a prospective period depending on the set values of the total turnover, price indices, and also on the set values of the parameter a_3 characterizing the influence of other factors on the formation of consumer demand of the population.

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