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Training And Research Studies Of Future Bachelor's Mathematicians During The Study Limits

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Abstract

The paper deals with issues of improving the effectiveness of professional training of bachelors in mathematics due to the intensification of their research activities by applying asymptotic expansions. In result, the non-effectiveness of applying the standard asymptotic expansions to them can be explained by the fact that their direct application does not allow to allocate in the numerators of the given fractions the main part which is not identically equal to zero. In conclusion, research activities are the resulting intellectual product that sets a certain truth in a particular subject area, new knowledge based on existing knowledge and skills.

Key Words: Research Work, Project Method, Mathematics.

Estudios de capacitación e investigación de futuros licenciados en matemáticas durante durante los límites del estudio

Resumen

El documento aborda temas relacionados con mejorar la efectividad de la capacitación profesional de licenciados en matemáticas debido a la intensificación de sus actividades de investigación mediante la aplicación de expansiones asintóticas. En consecuencia, la no efectividad de aplicarles las expansiones asintóticas estándar puede explicarse por el hecho de que su aplicación directa no permite asignar en los numeradores de las fracciones dadas la parte principal que no es idénticamente igual a cero. En conclusión, las actividades de investigación son el producto intelectual resultante que establece una cierta verdad en un área temática particular, nuevos conocimientos basados en los conocimientos y habilidades existentes.

Palabras Clave: Trabajo de Investigación, Método de Proyecto, Matemáticas.

1. INTRODUCTION

At present, the goals of education are focused on the formation and comprehensive development of creative, active student identity, the formation of students' abilities to independently acquire and apply knowledge, preparing them for subsequent employment and social activities. The variety of forms of scientific research works of students (SRWS) allows us to actively involve students in research activities, using new original methods. According to experts, development of science in higher education, its integration in the educational process

contributes to the quality of training, the formation of fully developed personality capable after graduation independently and responsibly to solve complex practical, scientific and management tasks (Matveeva, 2006; Jenaabadi & Shad, 2013).

According to the modern concept of the development of higher education, the use of techniques and methods during the training process, those form the students' ability to independently acquire new knowledge, to find, process and synthesize information, draw conclusions and make responsible decisions, is extremely actual. Therefore, SRWS is regarded not just as a way of getting new knowledge by the students, but also as a method of forming the necessary key, substantive and professional competences in future students. Moreover, special attention is paid to those forms of educational process organization of student's mathematical preparation, which contribute to the creative and intellectual development of the student's personality, his ability to self-education and self-development (Ahmadi et al, 2014; Nkeobuna, 2018; Fatihudin, 2019).

The study shows that scientific research activity of students is revealed fully exactly in the method of projects, which is considered by scientists as an integral component of productive training, as an effective stimulator of learning and cognitive activity of students (Polat, 2006). Method of projects, as pedagogical technology, provides a set of research, exploration and problem methods, promotes the independent informative activity of students in solving a specific problem and presenting its results in the form of a particular product (Polat, 2006; Soleimani et al., 2014). Saying the method of projects, we mean such technology of student's training, in which they are

directly involved in an active learning process, self-solve educational or research problems, collect the necessary information, develop ways to solve problems, find the best options for its solutions, analyze their activities, forming knowledge and acquiring life and learning experience (Kofanova, 2013; Mazana et al, 2019; Radhy, 2019).

2. RESULTS

Among the scientific research projects, a special place belongs to the educational and research projects. In particular, teaching and research projects held at the Yasawi International kazakh-turkish university, devoted to in-depth study of individual sections of the Mathematical Analysis course. Such projects include the performance of short- or long-term training or research projects by the students, writing articles and abstracts, presentations at conferences, participation in competitions, execution of projects and theses, etc. Below we present a teaching and research project that focuses on calculating limits, using standard asymptotic expansions (Bugrov&Nikolskiy, 1980; Rozhdestvenskiy, 1972; Safonov, 1989).

Project on the theme: Computation of limits, using standard asymptotic expansions. The aim of the project: Use of asymptotic expansions for calculating limits. Description of the project. Asymptotic expansions are widely used in both theoretical and applied research. They are used, for example, to simplify the rather complicated mathematical expressions. Apply asymptotic expansions should be cautious. Replacing one expression by another one, using

the asymptotic formula, may lead to a coarsening of the mathematical model. A simplified model can physically not adequately reflect the process, which is studied. Let us explain it with an example. The vibrations of the pendulum (if it neglects the friction) are described by the equation:

$$ml^2 \ddot{x} + mgl \sin x = 0 \quad (1)$$

Where m is a mass of pendulum; l is its length; g is an acceleration of gravity; x is the angle of deviation of the pendulum from the equilibrium position. Equation (1) is a non-linear differential equation. The problem is to find solutions of this equation, i.e. a function $x = x(t)$, which turns (1) into an identity. To find explicitly solution of (1) is not possible (due to non-linearity of $\sin x$). However, if in (1) we change $\sin x$ to x , then we get the equation:

$$ml^2 \ddot{x} + mgl x = 0 \quad (2)$$

Which already can be easily solved. It is easy to see that $x = x(t) = \sqrt{A^2 + B^2} \sin(\sqrt{g/l} \cdot t + \varphi_0)$, is a solution of (1), where the initial phase φ_0 is defined from the system of equalities:

$$\cos \varphi_0 = B / \sqrt{A^2 + B^2}, \sin \varphi_0 = A / \sqrt{A^2 + B^2} \quad (A, B - \text{const}).$$

Transition from the model (1) to a simple model (2) is possible for small values of x , since only in this case the following asymptotic formula holds:

$$\sin x = x + o(x) \quad (x \rightarrow 0) \quad (3)$$

where $o(x)$ is some function, which tends to zero faster than x . However, if x is not infinitesimal (e.g., $x = 0,9$), then the model (2) roughly approximates model (1) (in this case, the frequency ω and the oscillation of the pendulum (1) will essentially depend on the amplitude of the oscillations, while in (2) a constant frequency is $\omega = \sqrt{g/l}$). There are more accurate asymptotic formulas for $\sin x$. For example, if we use the asymptotic expansion:

$$\sin x = x - x^3/6 + o(x^4) \quad (x \rightarrow 0),$$

(4)

Then instead of (1) we obtain:

$$ml^2 \ddot{x} - x + x^3/6 = 0,$$

(5)

Which is a good approximation of model (1) with arguments x , that are not interrupt one radian. Thus, asymptotic formulas of the type (3) and (4) play an important role in the study of various physical processes. Using these formulas, equations, describing these processes can be essentially simplified. An important role is played information on the error of the asymptotic formulas. If the formula (3) is rough, then the simplified model (2), obtained with its help, is also rough; it does not adequately reflect the physical process. Conversely, if the asymptotic formula (3) is accurate, then the simplified model (4) - (5), obtained with its help, will be exact. General information. We formulate the theorem that establishes a connection between functions of the class $o(1)$ ($x \rightarrow x_0$) and functions, which have a limit at the point $x \rightarrow x_0$. Theorem 1. If there exists (finite) limit $\lim_{x \rightarrow x_0} f(x) = P$, then on

some neighborhood $U_{x_0}(\delta) \equiv \{0 < |x - x_0| < \delta\}$ of the point $x = x_0$ the function $f(x)$ can be represented in the following form:

$$f(x) = P + o(1)(x \rightarrow x_0).$$

(6)

Conversely: if on some neighborhood $U_{x_0}(\delta)$ of the point $x = x_0$ the function $f(x)$ can be represented as (6), then this function has a limit at the point $x = x_0$, moreover, $\lim_{x \rightarrow x_0} f(x) = P$.

Proof. Existence of the limit $(\lim_{x \rightarrow x_0} f(x) = P) \Leftrightarrow (f(x) = P + o(1)(x \rightarrow x_0))$ is equivalent to the statement:

$$\forall \varepsilon > 0 \quad \exists \delta = \delta(\varepsilon) > 0: (\forall x)(0 < |x - x_0| < \delta \Rightarrow |f(x) - P| < \varepsilon).$$

(7)

The statement (7) is equivalent to the fact that the function $\alpha(x) \equiv f(x) - P = o(1)(x \rightarrow x_0)$, or, the same $f(x) = P + o(1)(x \rightarrow x_0)$. Therefore, the statements $(\lim_{x \rightarrow x_0} f(x) = P)$ and $(f(x) = P + o(1)(x \rightarrow x_0))$ are equivalent to each other. Theorem is proved.

Let us now turn to the next fundamental concept in the theory of asymptotic expansions.

Definition. Representation of the function $f(x)$ on a deleted neighborhood of the point $x = x_0$ in the form:

$$f(x) = \Phi(x) + o(g(x)) (x \rightarrow x_0) \quad (8)$$

Is called asymptotic expansion of the function $f(x)$ (for the function $g(x)$) as $x \rightarrow x_0$. Moreover, the function $\Phi(x)$ is called the main part of the expansion (8) for the function $g(x)$, and the function $g(x)$ – pattern function. The simplest examples of asymptotic expansions are the expansion $f(x) = P + o(1) (x \rightarrow x_0)$ of the function $f(x)$, which has a limit P at the point $x = x_0$, and the expansion $f(x) = f(x_0) + o(1) (x \rightarrow x_0)$ of the function $f(x)$, continuous at the point $x = x_0$. In these expansions a pattern function is the function $g(x) \equiv 1$, and a main part $\Phi(x)$ – is the numbers P and $f(x_0)$, respectively. Of course, the asymptotic expansions as (8) are useful, in which the main part $\Phi(x)$ is easier than the original function $f(x)$. In this case it is convenient to use them in various calculations, for example, when calculating limits. We will show how it is done, without justification legality of expansions used below.

Example 1. Calculate the limit: $\lim_{x \rightarrow 0} \frac{\sin 3x}{\operatorname{tg} 2x} = P$.

Solution. Use the asymptotic expansions: $\sin u = u + o(u) (u \rightarrow 0)$, $\operatorname{tg} u = u + o(u) (u \rightarrow 0)$. In these expansions the main part $\Phi(u) \equiv u$ is much easier than the original functions $\sin u$ and $\operatorname{tg} u$. Using properties of the class $o(g(x)) (x \rightarrow x_0)$, we have:

$$\begin{aligned} \sin 3x &= 3x + o(3x) = 3x + o(O(1)x) = 3x + o(x), \\ \operatorname{tg} 2x &= 2x + o(2x) = 2x + o(O(1)x) = 2x + o(x) (x \rightarrow 0). \end{aligned}$$

Now it is easier to calculate the limit P . We have:

$$P = \lim_{x \rightarrow 0} \frac{3x + o(x)}{2x + o(x)} = \lim_{x \rightarrow 0} \frac{x(3 + o(1))}{x(2 + o(1))} = \lim_{x \rightarrow 0} \frac{3 + o(1)}{2 + o(1)} = \frac{3}{2}.$$

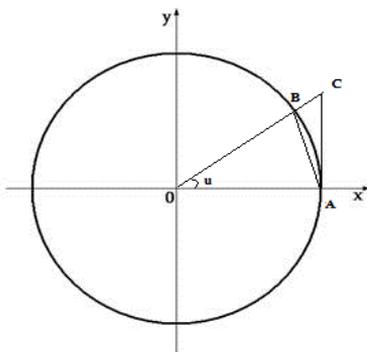
This example shows that for a successful calculation of limits it is necessary to have a stock of asymptotic expansions of basic elementary functions. Theorem 2. As $u \rightarrow 0$ the following asymptotic expansions hold:

- | | |
|---|---------------------------------------|
| 1) $\sin u = u + o(u),$ | 2) $\operatorname{tgu} = u + o(u),$ |
| 3) $\arcsin u = u + o(u),$ | 4) $\operatorname{arctgu} = u + o(u)$ |
| 5) $\cos u = 1 - u^2 / 2 + o(u^2),$ | 6) $e^u = 1 + u + o(u),$ |
| 7) $a^u = 1 + u \cdot \ln a + o(u), a > 0, a \neq 1,$ | 8) $\ln(1 + u) = u + o(u)$ |
| 9) $(1 + u)^\sigma = 1 + \sigma \cdot u + o(u), \sigma = \text{const.}$ | |

Proof. The first four expansions follow from the first remarkable limit $\lim_{u \rightarrow 0} (\sin u / u) = 1$. Check its justification. First prove the following statement:

$$(\forall u) (0 < |u| < \pi / 2 \Rightarrow |\sin u| < |u| < |\operatorname{tgu}|). \tag{9}$$

Since (9) is not changed if we place u by $-u$, then it is enough



to show (9) when $u \in \{0 < u < \pi / 2\}$. Consider the graph, where $|OA| = R$, $CA \perp OA$, $\angle COA = u$. It is clear, that area of the triangle OAB is less than the area of the sector OAB , and the area

of the sector is less than the area of the triangle OCA .

Calculating the corresponding areas, we write this fact by the inequalities:

$$\frac{1}{2}R^2 \sin u < \frac{1}{2}R^2 u < \frac{1}{2}R^2 \operatorname{tg} u \quad (|AC| = R \cdot \operatorname{tg} u),$$

or, dividing by $R^2/2$, we get the inequality $\sin u < u < \operatorname{tg} u$, which are equivalent to (9) when $u \in \{0 < u < \pi/2\}$. Justification of (9) for negative u , as we noted above, is not necessary. Let us now turn to the proof of the first remarkable limit. Dividing both sides of (9) to $|\sin u|$, we obtain the implication:

$$(\forall u) \left(0 < |u| < \frac{\pi}{2} \Rightarrow 1 < \left| \frac{u}{\sin u} \right| < \frac{1}{|\cos u|} \right).$$

Turning to the limit as $u \rightarrow 0$ and using continuity of the functions $\varphi(u) = \cos u$ and $f(x) = |x|$ (on the set R^1), we have:

$$\lim_{u \rightarrow 0} \left| \frac{u}{\sin u} \right| = 1 \Rightarrow \lim_{u \rightarrow 0} \left| \frac{\sin u}{u} \right| = \lim_{u \rightarrow 0} \frac{1}{|u \sin u|} = \frac{1}{1} = 1.$$

Taking into account that the function $\sin u$ and its argument u have one and the same sign for small u , we get that the last limit is equivalent to the first remarkable limit. Further arguments are based on Theorem 1:

$$\left(\lim_{u \rightarrow u_0} f(u) = P \right) \Leftrightarrow \left(f(u) = P + o(1) \quad (u \rightarrow u_0) \right).$$

(10)

Since the first remarkable limit holds, then, according to (10), the following asymptotic expansion is true:

$\frac{\sin u}{u} = 1 + o(1) \ (u \rightarrow 0) \Leftrightarrow \sin u = u + o(u) \ (u \rightarrow 0)$. Hence, the expansion 1 is proved. Further, since

$$\lim_{u \rightarrow 0} \frac{\operatorname{tgu}}{u} = \lim_{u \rightarrow 0} \frac{\sin u}{u} \cdot \lim_{u \rightarrow 0} \frac{1}{\cos u} = 1 \cdot 1 = 1,$$

Then again by Theorem 1, we have

$\frac{\operatorname{tgu}}{u} = 1 + o(1) \ (u \rightarrow 0) \Leftrightarrow \operatorname{tgu} = u + o(u) \ (u \rightarrow 0)$, which implies the expansion 2. Using the change of variables, $\arcsin u = x$, we find that

$$\lim_{u \rightarrow 0} \frac{\arcsin u}{u} = \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1, \quad \text{therefore (see (10)),}$$

$$\frac{\arcsin u}{u} = 1 + o(1) \ (u \rightarrow 0) \Leftrightarrow \arcsin u = u + o(u) \ (u \rightarrow 0)$$

The expansion 3 is also proved. Making the change, $\operatorname{arctg} u = x$, we

obtain that $\lim_{u \rightarrow 0} \frac{\operatorname{arctg} u}{u} = \lim_{x \rightarrow 0} \frac{x}{\operatorname{tg} x} = 1$, thus, by Theorem 1 we have

$$\frac{\operatorname{arctg} u}{u} = 1 + o(1) \ (u \rightarrow 0) \Leftrightarrow \operatorname{arctg} u = u + o(u) \ (u \rightarrow 0).$$

Expansion 4 is proved. Further, we have:

$$\cos u = \cos \left(2 \cdot \frac{u}{2} \right) = 1 - 2 \sin^2 \frac{u}{2}. \text{ Using the expansion 1, we obtain:}$$

$$\begin{aligned} \cos u &= 1 - 2 \left(\frac{u}{2} + o(u) \right)^2 = 1 - u^2 / 2 - 2u \cdot o(u) - 2o(u^2) = 1 - u^2 / 2 - o(u^2) \\ &= 1 - u^2 / 2 + o(u^2) \ (u \rightarrow 0), \end{aligned}$$

I.e. the expansion 5 is true. Rationale of the expansions 6-9are based on the second remarkable limit $\lim_{u \rightarrow 0} (1+u)^{1/u} = e$. Indeed, making the change of variables $e^u - 1 = x$ (then $u = \ln(1+x)$), we have:

$$\lim_{u \rightarrow 0} \frac{e^u - 1}{u} = \lim_{x \rightarrow 0} \frac{x}{\ln(1+x)} = \lim_{x \rightarrow 0} \frac{1}{\ln(1+x)^{1/x}} = \frac{1}{\ln e} = 1.$$

Using Theorem 1, we receive

$$\frac{e^u - 1}{u} = 1 + o(1) \quad (u \rightarrow 0) \quad \Leftrightarrow \quad e^u = 1 + u + o(u) \quad (u \rightarrow 0),$$

Which means that the expansion 6 is proved. Further, we have

$$a^u = e^{u \ln a} = 1 + u \ln a = o(u \ln a) = 1 + u \ln a + o(u) \quad (u \rightarrow 0),$$

That yields the expansion 7. Calculating the limit:

$$\lim_{u \rightarrow 0} \frac{\ln(1+u)}{u} = \lim_{u \rightarrow 0} \ln(1+u)^{1/u} = \ln e = 1$$

And using (10), we get:

$$\frac{\ln(1+u)}{u} = 1 + o(1) \quad (u \rightarrow 0) \quad \Leftrightarrow \quad \ln(1+u) = u + o(u) \quad (u \rightarrow 0),$$

i.e. the expansion 8 holds. At last, using the expansions 6 and 8, and properties of the class $o(g(x))$ ($x \rightarrow x_0$), we obtain:

$$(1+u)^\sigma \equiv e^{\sigma \ln(1+u)} = 1 + \sigma \ln(1+u) + o(\sigma \ln(1+u)) = 1 + \sigma(u + o(u)) + o(\sigma \cdot u + \sigma \cdot o(u)) = 1 + \sigma u + o(u) + o(u) = 1 + \sigma u + o(u) \quad (u \rightarrow 0),$$

Which implies the expansion 9. Theorem is completely proved. During the calculation of a limit as $x \rightarrow x_0$ it is recommended first by the direct passage to the limit as $x \rightarrow x_0$ to set the fact, will there be uncertainty in the calculation of this limit. If such a case does not

occur, then, in fact, this limit transition leads to the calculation of the limit. If there is uncertainty, it is recommended to write it in parentheses. This will mean that for the calculation of this limit it is required some transformation, the goal of which will consist of the elimination of uncertainty. For example, $P = \lim_{x \rightarrow 1} \frac{\sin \pi x}{x - 1} = \left(\frac{0}{0} \right) = \dots$

Further calculations should be made to ensure that the uncertainty has disappeared. This can be achieved, using standard asymptotic expansions 1 - 9 of Theorem 2. It should be noted that these expansions are valid only if the argument of the expanded function tends to zero. If not, we should first make a change of variables, which would lead to a new argument that tends to zero. Only then we can use asymptotic expansions. For example, in the above example, the argument πx of the function $\sin \pi x$ does not approaches to zero as $x \rightarrow 1$. Change variables: $u = x - 1$. Then Since $u = x - 1 \rightarrow 0$ as $x \rightarrow 1$, then the argument of the function $\sin \pi$ tends to zero. Now it is possible to apply the asymptotic expansions and begin to eliminate uncertainty. In order not to interrupt the recording, it is comfortable to write all the changes of variables and used asymptotic expansions in brackets between equality signs. For example, the limit P , mentioned above, will be calculated as follows:

$$P = \lim_{x \rightarrow 1} \frac{\sin \pi x}{x - 1} = \left(\frac{0}{0} \right) = [x - 1 = u],$$

$$x = u + 1 = \lim_{u \rightarrow 0} \frac{-\sin \pi u}{u} = [\sin \pi u = \pi u + o(u)(x \rightarrow 0)] = \lim_{u \rightarrow 0} \frac{-\pi u + o(u)}{u} =$$

The following formula arises often:

$$[g(x) + o(g(x))]^k = g^k(x) + o(g^k(x)) \quad x \rightarrow x_0$$

(11)

Which is true for any $k > 0$. Let us prove it. We have $[g(x) + o(g(x))]^k = g^k(x)[1 + o(1)]^k$.

Since as $x \rightarrow x_0$: $\lim_{x \rightarrow x_0} [1 + o(1)]^k = [1 + \lim_{x \rightarrow x_0} o(1)]^k = 1$, then due to Theorem1 the expansion $(1 + o(1))^k = 1 + o(1)$ $x \rightarrow x_0$, holds, therefore $[g(x) + o(g(x))]^k = g^k(x)(1 + o(1)) = g^k(x) + o(g^k(x))$ ($x \rightarrow x_0$).

Formula (11) is proved.

Example2. Calculate the limit: $P = \lim_{x \rightarrow 0} (\cos x)^{1/(1-3^x)}$.

Solution. If we pass directly to the limit as $x \rightarrow 0$, we get a power uncertainty of the type 1^∞ . Due to the remark, noted in the previous example, the calculation of the limit P is convenient to carry, turning to the exponential function

$$P = \lim_{x \rightarrow 0} e^{\frac{1}{1-3^x} \ln \cos x} = e^{\lim_{x \rightarrow 0} \frac{\ln \cos x}{1-3^x}}$$

Now the whole thing is reduced to the calculation of the limit on the exponent. We have:

$$P_1 = \lim_{x \rightarrow 0} \frac{\ln \cos x}{1-3^x} = \left(\frac{0}{0} \right) = \lim_{x \rightarrow 0} \frac{\ln(1 - x^2/2 + o(x^2))}{1 - (1 + x \ln 3 + o(x))} = \lim_{x \rightarrow 0} \frac{-x^2/2 + o(x^2) + o(-x^2/2)}{-x \ln 3 - o(x)}$$

$$= [o(g(x) + o(g(x))) = o(g(x))] = [\ln(1 + u) = u + o(u), \quad u = -x^2/2 + o(x^2)]$$

$$\begin{aligned}
 &= \lim_{x \rightarrow 0} \frac{-x^2/2 + o(x^2) + o(-x^2/2 + o(x^2))}{-x \ln 3 + o(x^2)} = \left[o(g(x) + o(g(x))) \right] = o(0) \\
 &= \lim_{x \rightarrow 0} \frac{-x^2/2 + o(x^2) + o(-x^2/2)}{-x \ln 3 + o(x)} = \lim_{x \rightarrow 0} \frac{-x^2/2 + o(x^2)}{-x \ln 3 + o(x)} = \lim_{x \rightarrow 0} \frac{x^2(-1/2)}{-x \ln 3}
 \end{aligned}$$

Consequently, the original limit P will be equal to $e^{P_1} = e^0 = 1$.

Consider an example where we want to calculate the limit of a function at infinity. Some artificial methods of obtaining asymptotic expansions of a higher order. We have demonstrated the feasibility of the method of asymptotic expansions on a wide set of examples. However, discussed examples do not cover all cases of limits that may be encountered in practice. To calculate some of them it is required more accurate asymptotic expansions than the expansions 1) - 9). Such, for example, are limits

$$\lim_{x \rightarrow 0} \frac{\sin x - x}{\operatorname{tg}^3 x}, \quad \lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2}.$$

The non-effectiveness of applying the standard asymptotic expansions to them can be explained by the fact that their direct application does not allow to allocate in the numerators of the given fractions the main part which is not identically equal to zero. We need more precise asymptotic expansions. There is a general method for the asymptotic expansions of higher orders.

Example 3. Find an asymptotic expansion of the second order for the function $f(x) = \ln(1+x)$ concerning to the function $g(x) = x$ as $x \rightarrow 0$.
 Solution. It is necessary to obtain an asymptotic expansion of the form:

$$\ln(1+x) = a_0 + a_1x + a_2x^2 + o(x^2) \quad (x \rightarrow 0),$$

$$(12)$$

Where the coefficients a_j are required to define. Using uniqueness of the main parts of power asymptotic expansions and standard asymptotic expansion $\ln(1+x) = x + o(x)$ ($x \rightarrow 0$), we find that $a_0 = 0$, $a_1 = 1$. consequently, (12) has the form:

$$\ln(1+x) = a_0 + a_2x^2 + o(x^2) \quad (x \rightarrow 0).$$

$$(13)$$

Applying the expansion (13), we get:

$$\begin{aligned} \ln(1+x^2+2x) &= \left[\ln(1+u) = u + a_2u^2 + o(u^2), u = x^2 + 2x \rightarrow 0 \right] = \\ &= (x^2 + 2x) + a_2(x^2 + 2x) + o((x^2 + 2x)^2) = 2x + (4a_2 + 1)x^2 + o(x^2) \end{aligned}$$

On the other hand, the formula (13) implies:

$$\ln(1+x^2+2x) \equiv \ln(1+x)^2 = 2\ln(1+x) = 2(x + a_2x^2 + o(x^2)) = 2x + 2a_2x^2 + o(x^2)$$

Comparing this expansion with the previous expansion, we find that $4a_2 + 1 = 2a_2$, i.e. $a_2 = -1/2$.

Thus, the expansion (13) takes the form:

$$\ln(1+x) = x - x^2/2 + o(x^2) \quad (x \rightarrow 0).$$

3. CONCLUSION

During the project - research activity of students different competencies are formed: reflective; search; communication;

presentation; general training (working with a textbook and scientific literature, preparation of tables, expression of thought, implementation of self-control and self-analysis); special (assimilation of factual material); research competence (ability to formulate the goal, to make a plan and program of research, to describe the phenomenon, to formulate a hypothesis, to integrate data, to use mathematical tools to describe the laws and regularities, to draw conclusions). In general, research competences characterize the willingness of students to the implementation of research activities on the basis of knowledge and life experience, with knowledge of the aim, conditions and means of activities, aimed at the study and elucidation of the processes, facts and phenomena. The main conclusion of the project - research activities are the resulting intellectual product that sets a certain truth in a particular subject area, new knowledge, which is based on existing knowledge and skills and is represented in the research work of the student. The result of the project - research work is made by writing. It can be presented as a report, essay, project, coursework and thesis. This is, in fact, a product of project - research activities.

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