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Some new sequence spaces of interval number based on Zweier sequences and Fibonacci numbers

Algunos nuevos espacios de secuencia de número de intervalo basados en secuencias de Zweier y números de Fibonacci

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Abstract

The main aim of this paper is to determine necessary and sufficient conditions for the matrix of interval numbers $\bar{A} = (\bar{a}_{nk})$ such that $\bar{A}$ transform $\bar{x} = (\bar{x}_k)$, belongs to the set l^i_∞, c^i_0, c^i where in particular $\bar{x} \in l^i_\infty$, c^i_0 and c^i, to introduce some new sequence spaces $c^i(\bar{A}_{ZF})$, $c^i_0(\bar{A}_{ZF})$, $l^i_\infty(\bar{A}_{ZF})$ based on a newly defined matrix of interval numbers $\bar{A}_{ZF}$. We study some basic algebraic and topological properties. Also we investigate the relations related to these spaces.

Key words and phrases: Matrix transformations, interval number, Zweier sequence, Fibonacci number.

Resumen

El objetivo principal de este documento es determinar condiciones necesarias y suficientes para la matriz de números de intervalos $\bar{A} = (\bar{a}_{nk})$ tal que $\bar{A}$ transforma $\bar{x} = (\bar{x}_k)$, perteneciente al conjunto l^i_∞, c^i_0, c^i donde en particular $\bar{x} \in l^i_\infty, c^i_0$ y c^i, para introducir algunos nuevos espacios de secuencia $c^i(\bar{A}_{ZF})$, $c^i_0(\bar{A}_{ZF})$, $l^i_\infty(\bar{A}_{ZF})$ basado en una matriz de números de intervalo $\bar{A}_{ZF}$, definida nuevamente. Nosotros estudiamos algunas propiedades algebraicas y topológicas básicas. También investigamos las relaciones referentes a estos espacios.

Palabras y frases clave: Transformaciones de matriz, número de intervalo, Zweier secuencia, número de Fibonacci.

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1 Introduction

The sequence of interval numbers and usual convergence of sequences of interval numbers are studied by Chiao [4]. Later, Sengonul and Eryilmaz [28] introduced and studied bounded and convergent sequence spaces of interval numbers and showed that these spaces are complete metric space. In the recent days, Esi [20] introduced and studied strongly almost λ-convergence and statistically almost λ-convergence of interval numbers and lacunary sequence spaces of interval numbers, respectively. For more information about interval numbers one may refer to Debnath et. al. [5, 7, 8, 9], Dwyer [10, 11], Fischer [22], Moore [26], Moore and Yang [25], Esi [12]–[17].

The theory of matrix transformations is a wide field in summability; it deals with the characterizations of classes of matrix mappings between sequence spaces by giving necessary and sufficient conditions on the entries of the infinite matrices. Matrix transformations in sequence spaces have been studied by different authors like Tripathy [30, 31], Savas and Mursaleen [27] and many others.

2 Preliminaries

An interval number $\bar{x}$ is a closed subset of the real numbers and denoted as $\bar{x} = [x_l, x_r]$, where $x_l \leq x_r$ and $x_l, x_r \in \mathbb{R}$. Let us denote the set of real valued closed intervals by $R(I)$. The absolute value (magnitude or interval norm) of an interval number is defined by

$$|\bar{x}| = \max\{|x_l|, |x_r|\}.$$

In general, ordinary distributive law does not hold for interval arithmetic. If $\bar{x}$, $\bar{y}$, $\bar{z}$ are any three intervals then it is easy to verify that

$$\bar{x}.(\bar{y}+\bar{z}) \subseteq \bar{x}.\bar{y} + \bar{x}.\bar{z}$$
$$|\bar{x}.(\bar{y}+\bar{z})| \leq |\bar{x}.\bar{y}| + |\bar{x}.\bar{z}|$$
$$|\bar{x}.\bar{y}| \leq |\bar{x}|.|\bar{y}|$$

For $\bar{x}_1, \bar{x}_2 \in R(I)$, we have $\bar{x}_1 = \bar{x}_2$ iff $x_{l1} = x_{l2}$, $x_{r1} = x_{r2}$;

$$\bar{x}_1 + \bar{x}_2 = \{x \in \mathbb{R} : x_{l1} + x_{l2} \leq x \leq x_{r1} + x_{r2}\},$$

If $\alpha \geq 0$, then

$$\alpha\bar{x} = \{x \in \mathbb{R} : \alpha x_l \leq x \leq \alpha x_r\}$$

and if $\alpha < 0$, then

$$\alpha\bar{x} = \{x \in \mathbb{R} : \alpha x_r \leq x \leq \alpha x_l\}$$

Let us defined

$$\alpha = \min\{x_{l1}.x_{l2},\ x_{l1}.x_{r2},\ x_{r1}.x_{l2},\ x_{r1}.x_{r2}\} \text{ and } \beta = \max\{x_{l1}.x_{l2},\ x_{l1}.x_{r2},\ x_{r1}.x_{l2},\ x_{r1}.x_{r2}\}$$

So

$$\bar{x}_1.\bar{x}_2 = \{x \in \mathbb{R} : \alpha \leq x \leq \beta\}.$$

The set of all interval numbers $R(I)$ is a complete metric space defined by

$$d(\bar{x}_1, \bar{x}_2) = \max\{|x_{l1} - x_{l2}|, |x_{r1} - x_{r2}|\}.$$

In the special case $\bar{x}_1 = [a, a]$ and $\bar{x}_2 = [b, b]$, we obtain usual metric on the $\mathbb{R}$ with

$$d(\bar{x}_1, \bar{x}_2) = |a - b|.$$

Let us define the transformation f from $\mathbb{N}$ to $R(I)$ by k to $f(k) = \bar{x}$, $\bar{x} = (\bar{x}_k)$. Then $(\bar{x}_k)$ is called sequence of interval numbers. The $\bar{x}_k$ is called k^{th} term of sequence $(\bar{x}_k)$.

Definition 2.1. A sequence $\bar{x} = (\bar{x}_k)$ of interval numbers is said to be convergent to the interval number $\bar{x}_0$ if for each $\epsilon > 0$ there exists a positive number k_0 such that $d(\bar{x}_k, \bar{x}_0) < \epsilon$ for all $k \geq k_0$ and we denote it by $\lim_k \bar{x}_k = \bar{x}_0$. Thus $\lim_k \bar{x}_k = \bar{x}_0$, $\lim_k x_{lk} = \lim_k x_{l0}$ and $\lim_k x_{rk} = \lim_k x_{r0}$.

Definition 2.2. An interval valued sequence space $\bar{E}$ is said to be solid if $\bar{y} = (\bar{y}_k) \in \bar{E}$ whenever $|\bar{y}_k| \leq |\bar{x}_k|$ for all $k \in \mathbb{N}$ and $\bar{x} = (\bar{x}_k) \in \bar{E}$.

Through out this paper w^i, l^i_∞, c^i and c^i_0 denote the set of all, bounded, convergent and null sequences of interval numbers with real terms. We denote $\bar{e}_k$ as the interval sequence whose k^{th} term is $[1,1]$ and the other terms are $[0,0]$ and $(\bar{e}) = ([1,1], [1,1], [1,1], \ldots)$.

Let $\bar{X}, \bar{Y}$ be two sequence spaces of interval numbers and let $\bar{A} = (\bar{a}_{nk})$ be an infinite matrix of interval numbers $\bar{a}_{nk}$, where $n, k \in \mathbb{N}$.

Then the matrix $\bar{A}$ defines the $\bar{A}$-transformation from X into Y, if for every sequence $\bar{x} = (\bar{x}_k) \in \bar{X}$, the sequence $\bar{A}\bar{x} = \{(\bar{A}\bar{x})_n\}$, the $\bar{A}$-transform of $\bar{x}$ exists and is in $\bar{Y}$; where $(\bar{A}\bar{x})_n = \sum_k \bar{a}_{nk}\bar{x}_k$. For simplicity in notation, here and in what follows, the summation without limits runs from 0 to ∞. By $\bar{A}(\bar{X} : \bar{Y})$ we mean the characterizations of matrices from $\bar{X}$ to $\bar{Y}$ i.e., $\bar{A} : \bar{X} \to \bar{Y}$. A sequence $\bar{x}$ is said to be $\bar{A}$-summable to $\bar{x}_0$ if $\bar{A}\bar{x}$ converges to $\bar{x}_0$ which is called as the $\bar{A}$-limit of $\bar{x}$.

For the sequence space $\bar{X}$, the matrix domain $\bar{X}_{\bar{A}}$ of an infinite matrix of interval numbers $\bar{A}$ is defined as

$$\bar{X}_{\bar{A}} = \{\bar{x} = (\bar{x}_k) : \bar{A}\bar{x} \in \bar{X}\}$$

Recently, the authors have introduced that, $\bar{A} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}]) \in (l^i_\infty : l^i_\infty)$ if and only if

$$\sup_{n \in \mathbb{N}} \sum_k |\bar{a}_{nk}| < \infty$$

3 Main Results

Theorem 3.1. $\bar{A} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}]) \in \left(c^i : c^i\right)$ *if and only if*

(i) $\sup \sum_k |\bar{a}_{nk}| < \infty$

(ii) $\bar{a}_k = \lim_{n \to \infty} \bar{a}_{nk}$ *exists for each* $k \in \mathbb{N}$.

(iii) $\lim_{n \to \infty} \sum_k \bar{a}_{nk} = \bar{a} = [a_l, a_r]$ *where* $a_l, a_r \in \mathbb{C}$

Proof. The necessity of *(i)* follows from the inclusion relation $(c^i : c^i) \subseteq (c^i : l^i_\infty)$. That of *(ii)* and *(iii)* follows on considering the sequence $\bar{x} = (\bar{e}_k)$ and $\bar{x} = (\bar{e})$, respectively.

Conversely, suppose that the conditions *(i)*, *(ii)* and *(iii)* hold and $\bar{x} = (\bar{x}_k) \in c^i$ with $\bar{x}_k \to \bar{x}_0$ as $k \to \infty$, i.e., $x_{lk} \to x_{l0}$ and $x_{rk} \to x_{r0}$ as $k \to \infty$.

Now,

$$\sum_k a_{lnk} x_{lk} = \sum_k a_{lnk}(x_{lk} - x_{l0}) + x_{l0} \sum_k a_{lnk} \tag{3.1}$$

and

$$\sum_k a_{rnk} x_{rk} = \sum_k a_{rnk}(x_{rk} - x_{r0}) + x_{r0} \sum_k a_{rnk} \tag{3.2}$$

holds for each $n \in \mathbb{N}$.

In equations (3.1) and (3.2), since the first terms of the right hand side tends to

$$\sum_k a_{lk}(x_{lk} - x_{l0}) \text{ and} \sum_k a_{rk}(x_{rk} - x_{r0})$$

respectively by *(ii)*, and the second terms on the right hand side tends to $x_{l0}a_l$ and $x_{r0}a_r$ respectively by *(iii)*, as $n \to \infty$, hence

$$\lim_{n\to\infty} \left(\sum_k a_{lnk} x_{lk} \right) = \sum_k a_{lnk}(x_{lk} - x_{l0}) + x_{l0} a_l$$

$$\lim_{n\to\infty} \left(\sum_k a_{rnk} x_{rk} \right) = \sum_k a_{rnk}(x_{rk} - x_{r0}) + x_{r0} a_r.$$

Hence $\bar{A}\bar{x} \in c^i$. □

Corollary 3.1. $\bar{A} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}]) \in (c^i : c_0^i)$ *if and only if*

(i) $\sup_{n\in\mathbb{N}} \sum_k |\bar{a}_{nk}| < \infty$.

(ii) $\bar{a}_k = \lim_{n\to\infty} \bar{a}_{nk}$ *exists for each* $k \in \mathbb{N}$.

(iii) $\lim_{n\to\infty} \sum_k \bar{a}_{nk} = \theta$, *where* $\theta = [0, 0]$.

Theorem 3.2. $\bar{A} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}]) \in (c_0^i : c^i)$ *if and only if*

(i) $\sup_{n\in\mathbb{N}} \sum_k |\bar{a}_{nk}| < \infty$.

(ii) $\bar{a}_k = \lim_{n\to\infty} \bar{a}_{nk}$ *exists for each* $k \in \mathbb{N}$.

Proof. The necessity of *(i)* follows from the inclusion relation $(c_0^i : c^i) \subseteq (c_0^i : l_\infty^i)$. That of *(ii)* follows on considering the sequence $\bar{x} = (\bar{e}_k)$.

Conversely, assume that the conditions *(i)*, *(ii)* hold and $\bar{x} = (\bar{x}_k) \in c_0^i$. So that $\bar{x}_k \to \theta$ as $k \to \infty$, where $\theta = [0, 0]$, i.e., $x_{lk} \to 0$ and $x_{rk} \to 0$ as $k \to \infty$.

Now, $\sum_k a_{lnk} x_{lk}$ tends to $\sum_k a_{lk} x_{lk}$ as $n \to \infty$ and $\sum_k a_{rnk} x_{rk}$ tends to $\sum_k a_{rk} x_{rk}$ as $n \to \infty$. Hence $\bar{A}\bar{x} \in c^i$. □

Corollary 3.2. $\bar{A} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}]) \in (c_0^i : c_0^i)$ *if and only if*

(i) $\sup_{n \in N} \sum_k |\bar{a}_{nk}| < \infty.$

(ii) $\lim_{n \to \infty} \bar{a}_{nk} = \theta$ *exists for each* $k \in \mathbb{N}$, *where* $\theta = [0, 0]$.

The approach of constructing new sequence spaces by means of the matrix domain of a particular limitation method have recently been employed by Altay, Basar and Mursaleen [1], Basar and Altay [2], Malkowsky [24] and many others.

Sengonul [29] defined the sequence $y = (y_i)$ which is frequently used as the Z^p transform of the sequence $x = (x_i)$, i.e.,

$$y_i = px_i + (1 - p)x_{i-1}$$

where $x_{-1} =$, $1 < p < \infty$ and Z^p denotes the matrix $Z^p = (Z_{ik})$ defined by

$$Z_{ik} = \begin{cases} p & i = k \\ 1 - p & i - 1 = k \quad \text{where } (i, k \in \mathbb{N}) \\ 0 & \text{otherwise} \end{cases} \tag{3.3}$$

Following Basar and Atlay [2], Sengonul [29] introduced the Zweier sequence spaces Z and Z_0 as follows:

$$Z = \{x = (x_k) \in w : Z^p x \in c\}$$
$$Z_0 = \{x = (x_k) \in w : Z^p x \in c_0\}.$$

Furthermore, Esi and others [6, 14, 15], have developed Zweier sequence spaces in different directions. Recently, Debnath et. al. [8] have established the conditions of regularity of a matrix of interval numbers and introduced a regular matrix of interval numbers using Fibonacci numbers.

Now, we have constructed a matrix of interval numbers $\bar{A}_{ZF} = (\bar{a}_{nk}) = ([a_{lnk}, a_{rnk}])$, where using a_{lnk} are from Zweier sequences and a_{rnk} are from Fibonacci numbers.

To construct the matrix of interval numbers we put $p = \frac{1}{2}$ in (3.3).

$$a_{lnk} = \begin{cases} \frac{1}{2}, & n = k; \\ \frac{1}{2}, & n - 1 = k; \\ 0, & otherwise. \end{cases} \qquad \text{and} \qquad a_{rnk} = \begin{cases} \dfrac{f_k}{f_{n+2} - 1}, & 1 \le k \le n; \\ 0, & \text{otherwise.} \end{cases}$$

where, f_k is the Fibonacci numbers such that $f_k = f_{k-1} + f_{k-2}$ that is,

$$\bar{A}_{ZF} = \begin{pmatrix} [\frac{1}{2}, 1] & [0,0] & [0,0] & [0,0] & [0,0] & [0,0] & [0,0] & \dots & \dots & \dots\dots \\ [\frac{1}{2}, \frac{1}{2}] & [\frac{1}{2}, \frac{1}{2}] & [0,0] & [0,0] & [0,0] & [0,0] & [0,0] & \dots & \dots & \dots\dots \\ [0, \frac{1}{4}] & [\frac{1}{2}, \frac{1}{4}] & [\frac{1}{2}, \frac{2}{4}] & [0,0] & [0,0] & [0,0] & [0,0] & \dots & \dots & \dots\dots \\ [0, \frac{1}{7}] & [0, \frac{1}{7}] & [\frac{1}{2}, \frac{2}{7}] & [\frac{1}{2}, \frac{3}{7}] & [0,0] & [0,0] & [0,0] & \dots & \dots & \dots\dots \\ [0, \frac{1}{12}] & [0, \frac{1}{12}] & [0, \frac{2}{12}] & [\frac{1}{2}, \frac{3}{12}] & [\frac{1}{2}, \frac{5}{12}] & [0,0] & [0,0] & \dots & \dots & \dots\dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots\dots \end{pmatrix}$$

It is obvious that the above matrix is regular [8]. Now, we introduce the following sequence spaces based on infinite matrix $\bar{A}_{ZF}$

$$c^i(\bar{A}_{ZF}) = \{\bar{x} = (\bar{x}_k) \in w^i : \bar{A}_{ZF}\bar{x} \in c^i\}$$

$$c_0^i(\bar{A}_{ZF}) = \{\bar{x} = (\bar{x}_k) \in w^i : \bar{A}_{ZF}\bar{x} \in c_0^i\}$$

$$l_\infty^i(\bar{A}_{ZF}) = \{\bar{x} = (\bar{x}_k) \in w^i : \bar{A}_{ZF}\bar{x} \in l_\infty^i\}$$

where $\bar{A}_{ZF}\bar{x} = \{(\bar{A}_{ZF}\bar{x})_n\}$ and

$$\bar{A}_{ZF_n}(\bar{x}) = \sum_{k=1}^{\infty} \bar{a}_{nk}(x_{lk}, x_{rk}), \ (n \in \mathbb{N}).$$

Theorem 3.3. *$c^i(\bar{A}_{ZF})$ and $c_0^i(\bar{A}_{ZF})$ are metric spaces with the metric defined by*

$$d(\bar{x}_k, \bar{y}_k) = \sup_k \{\max\{|x_{lk} - y_{lk}|, |x_{rk} - y_{rk}|\}\}.$$

Proof. Easy, so omitted. □

Theorem 3.4. *The spaces $c_0^i(\bar{A}_{ZF})$, $c^i(\bar{A}_{ZF})$ and $l_\infty^i(\bar{A}_{ZF})$ are normed interval spaces with the norm*

$$\|\bar{x}\| = \sup_k \{\max\{|x_{lk}|, |x_{rk}|\}\}.$$

Proof. Let $\mu^i = c_0^i(\bar{A}_{ZF})$ (or $c^i(\bar{A}_{ZF})$ or $l_\infty^i(\bar{A}_{ZF})$) and $\bar{x}$, $\bar{y} \in \mu^i$.

N1. Since $\|\bar{x}\|_{\mu^i} = \sup_k\{\max\{|x_{lk}|, |x_{rk}|\}\}$, then we have $\|\bar{x}\|_{\mu^i} > 0, \forall \bar{x} \in \mu^i - \{\theta\}$.

N2. $\|\bar{x}\|_{\mu^i} = 0 \Leftrightarrow \sup_k\{\max\{|x_{lk}|, |x_{rk}|\}\} = 0 \Leftrightarrow \bar{x} = \theta$.

N3. $\|\bar{x} + \bar{y}\|_{\mu^i} = \sup_k\{\max\{|x_{lk} + y_{lk}|, |x_{rk} + y_{rk}|\}\} \leq \sup_k\{\max\{|x_{lk}| + |y_{lk}|, |x_{rk}| + |y_{rk}|\}\}$

$\leq \sup_k\{\max\{(|x_{lk}|, |x_{rk}|)\}\} + \sup_k\{\max\{(|y_{lk}|, |y_{rk}|)\}\} = \|\bar{x}\|_{\mu^i} + \|\bar{y}\|_{\mu^i}$.

N4. $\|\alpha\bar{x}\|_{\mu^i} = \sup_k\{\max\{|\alpha x_{lk}|, |\alpha x_{rk}|\}\} = |\alpha| \sup_k\{\max\{|x_{lk}|, |x_{rk}|\}\} = |\alpha|\|\bar{x}\|_{\mu^i}$.

Hence, $\|\bar{x}\|_{\mu^i}$ is a norm on μ^i. □

Theorem 3.5. *The spaces $c_0^i(\bar{A}_{ZF})$ and $c^i(\bar{A}_{ZF})$ are solid.*

Proof. We consider only $c_0^i(\bar{A}_{ZF})$. Let $\|\bar{y}_k\| \leq \|\bar{x}_k\|$, for all $(k \in \mathbb{N})$ and for some $\bar{x} \in c_0^i(\bar{A}_{ZF})$. Then clearly $\widetilde{d}(\bar{y}_k, \theta) \leq \widetilde{d}(\bar{x}_k, \theta)$, i.e., $\{|y_{lk} - 0|, |y_{rk} - 0|\} \leq \{|x_{lk} - 0|, |x_{rk} - 0|\}$. Thus, we have $y_{lk} \leq x_{lk}$ and $y_{rk} \leq x_{rk}$, i.e, $\bar{y} \leq \bar{x}$. So, $\bar{y} \in c_0^i(\bar{A}_{ZF})$. Hence, $c_0^i(\bar{A}_{ZF})$ is solid or normal.

For the space $c^i(\bar{A}_{ZF})$, the result can be proved similarly. □

Theorem 3.6.

(i) The inclusion $c_0^i \subset c_0^i(\bar{A}_{ZF})$ strictly holds.

(ii) The inclusion $c^i \subset c^i(\bar{A}_{ZF})$ strictly holds.

Proof. (i) Let, $\bar{x} = (\bar{x}_k) \in c_0^i$ then clearly $\lim_{k\to\infty} \bar{x}_k = \theta$, where $\theta = [0,0]$. Again since $\bar{A}_{ZF}$ is regular so $\bar{A}_{ZF}\bar{x} \in c_0^i$, i.e., $\bar{x} = (\bar{x}_k) \in c_0^i(\bar{A}_{ZF})$. Now, let $\bar{x} = (\bar{x}_k) = ([(-1)^k, \frac{1}{k}])$, $k \in \mathbb{N}$, then $\bar{x} = (\bar{x}_k) \in c_0^i(\bar{A}_{ZF})$ but $(\bar{x}_k)$ does not belongs to c_0^i.

(ii) Let $\bar{x} = (\bar{x}_k) \in c^i$. Again, since $\bar{A}_{ZF}$ is regular so $\bar{A}_{ZF}\bar{x} \in c_0^i$, i.e., $(\bar{x}_k) \in c^i(\bar{A}_{ZF})$. Now, let $\bar{x} = (\bar{x}_k)$, where

$$\bar{x}_k = \begin{cases} [1, 1 + \dfrac{1}{k}], & k \text{ odd} \\ [0, 1 + \dfrac{1}{k}], & k \text{ even} \end{cases}$$

then $\bar{x} = (\bar{x}_k) \in c^i(\bar{A}_{ZF})$ but $(\bar{x}_k)$ does not belongs to c^i. □

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