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On contra $e^*\theta$-continuous functions

En funciones contra $e^*\theta$-continuas

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Abstract

The main goal of this paper is to introduce and study a new type of contra continuity called contra $e^*\theta$-continuity. Also, we obtain fundamental properties and several characterizations of contra $e^*\theta$-continuous functions via e^*-θ-closed sets which are defined by Farhan and Yang [11]. Moreover, we investigate the relationships between contra $e^*\theta$-continuous functions and other related generalized forms of contra continuity.

Key words and phrases: e^*-θ-open set, e^*-θ-closed set, contra $e^*\theta$-continuity, $e^*\theta$-continuity, contra $e^*\theta$-closed graph.

Resumen

El objetivo principal de este documento es presentar y estudiar un nuevo tipo de contra continuidad llamada contra $e^*\theta$-continuidad. Además, obtenemos propiedades fundamentales y varias caracterizaciones de funciones contra $e^*\theta$-continuas a través de conjuntos e^*-θ cerrados que están definidos por Farhan y Yang [11]. Además, investigamos las relaciones entre las funciones contra continuas y otras formas generalizadas relacionadas de $e^*\theta$-continuidad de contra.

Palabras y frases clave: e^*-θ-conjunto abierto, e^*-θ-conjunto cerrado, contra $e^*\theta$-continuidad, $e^*\theta$-continuidad, contra $e^*\theta$-gráfico cerrado.

1 Introduction

In 1996, the concept of contra continuity [6], which is stronger than contra α-continuity [12], contra precontinuity [13], contra semicontinuity [7], contra b-continuity [17], contra β-continuity [5], is defined by Dontchev. Many results have been obtained related to the notions mentioned above recently. In this paper, we define and study the notion of contra $e^*\theta$-continuity which is stronger than contra e^*-continuity [10] and weaker than contra $\beta\theta$-continuity [4]. Also, we obtain several characterizations of contra $e^*\theta$-continuous functions and investigate their some fundamental properties. Moreover, we investigate the relationships between contra $e^*\theta$-continuous functions and seperation axioms and contra $e^*\theta$-closedness of graphs of functions.

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2 Preliminaries

Throughout this present paper, X and Y represent topological spaces. For a subset A of a space X, $cl(A)$ and $int(A)$ denote the closure of A and the interior of A, respectively. The family of all closed (resp. open, clopen) sets of X is denoted $C(X)$(resp. $O(X), CO(X)$). A subset A is said to be regular open [23] (resp. regular closed [23]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point $x \in X$ is said to be δ-cluster point [24] of A if $int(cl(U)) \cap A \neq \emptyset$ for each open neighbourhood U of x. The set of all δ-cluster points of A is called the δ-closure [24] of A and is denoted by $cl_\delta(A)$. If $A = cl_\delta(A)$, then A is called δ-closed [24], and the complement of a δ-closed set is called δ-open [24]. The set $\{x|(U \in O(X,x))(int(cl(U)) \subseteq A)\}$ is called the δ-interior of A and is denoted by $int_\delta(A)$.

A subset A is called α-open [18] (resp. semiopen [14], preopen [15], b-open [2], β-open [1], e-open [8], e^*-open [9]) if $A \subseteq int(cl(int(A)))$ (resp. $A \subseteq cl(int(A))$, $A \subseteq int(cl(A))$, $A \subseteq int(cl_\delta(A))$, $A \subseteq cl(int(A)) \cup int(cl(A))$, $A \subseteq cl(int(cl(A)))$, $A \subseteq cl(int_\delta(A)) \cup int(cl_\delta(A))$, $A \subseteq cl(int(cl_\delta(A)))$). The complement of an α-open (resp. semiopen, preopen, b-open, β-open, e-open, e^*-open) set is called α-closed [18] (resp. semiclosed [14], preclosed [15], b-closed [2], β-open [1], e-closed [8], e^*-closed [9]). The intersection of all e^*-closed (resp. semi-closed, pre-closed) sets of X containing A is called the e^*-closure [9] (resp. semi-closure [14], pre-closure [15]) of A and is denoted by e^*-$cl(A)$ (resp. $scl(A)$, $pcl(A)$). The union of all e^*-open (resp. semiopen, preopen) sets of X contained in A is called the e^*-interior [9] (resp. semi-interior [14], pre-interior [15]) of A and is denoted by e^*-$int(A)$ (resp. $sint(A)$, $pint(A)$).

The union of all e^*-open sets of X contained in A is called the e^*-interior [9] of A and is denoted by e^*-$int(A)$. A subset A is said to be e^*-regular [11] if it is e^*-open and e^*-closed. The family of all e^*-regular subsets of X is denoted by $e^*R(X)$.

A point x of X is called an e^*-θ-cluster (β-θ-cluster) point of A if e^*-$cl(U) \cap A \neq \emptyset$ for every e^*-open (resp. β-open) set U containing x. The set of all e^*-θ-cluster (β-θ-cluster) points of A is called the e^*-θ-closure [11] (β-θ-closure [19]) of A and is denoted by e^*-$cl_\theta(A)$ (β-$cl_\theta(A)$). A subset A is said to be e^*-θ-closed [11] (β-θ-closed [19]) if $A = e^*$-$cl_\theta(A)$ ($A = \beta$-$cl_\theta(A)$). The complement of an e^*-θ-closed (β-θ-closed) set is called an e^*-θ-open [11] (β-θ-open [19]) set. A point x of X is said to be an e^*-θ-interior [11] (β-θ-interior [19]) point of a subset A, denoted by e^*-$int_\theta(A)$ (β-$int_\theta(A)$), if there exists an e^*-open (β-open) set U of X containing x such that e^*-$cl(U) \subseteq A$ (β-$cl(U) \subseteq A$). Also it is noted in [11] that

$$e^*\text{-regular} \Rightarrow e^*\text{-}\theta\text{-open} \Rightarrow e^*\text{-open}.$$

The family of all open (resp. closed, e^*-θ-open, e^*-θ-closed, e^*-open, e^*-closed, regular open, regular closed, δ-open, δ-closed, semiopen, semiclosed, preopen, preclosed) subsets of X is denoted by $O(X)$ (resp. $C(X)$, $e^*\theta O(X)$, $e^*\theta C(X)$, $e^*O(X)$, $e^*C(X)$, $RO(X)$, $RC(X)$, $\delta O(X)$, $\delta C(X)$, $SO(X)$, $SC(X)$, $PO(X)$, $PC(X)$). The family of all open (resp. closed, e^*-θ-open, e^*-θ-closed, e^*-open, e^*-closed, regular open, regular closed, δ-open, δ-closed, semiopen, semiclosed, preopen, preclosed) sets of X containing a point x of X is denoted by $O(X,x)$ (resp. $C(X,x)$, $e^*\theta O(X,x)$, $e^*\theta C(X,x)$, $e^*O(X,x)$, $e^*C(X,x)$, $RO(X,x)$, $RC(X,x)$, $\delta O(X,x)$, $\delta C(X,x)$, $SO(X,x)$, $SC(X,x)$, $PO(X,x)$, $PC(X,x)$).

We shall use the well-known accepted language almost in the whole of the proofs of the theorems in this article. The following basic properties of e^*-closure and e^*-interior are useful in the sequel:

Lemma 2.1. *[9] Let A be a subset of a space X, then the following hold:*

(1) $e^*\text{-}cl(X \setminus A) = X \setminus e^*\text{-}int(A)$.

(2) $x \in e^*\text{-}cl(A)$ *if and only if* $A \cap U \neq \emptyset$ *for every* $U \in e^*O(X, x)$.

(3) A *is* $e^*C(X)$ *if and only if* $A = e^*\text{-}cl(A)$.

(4) $e^*\text{-}cl(A) \in e^*C(X)$.

(5) $e^*\text{-}int(A) = A \cap cl(int(cl_\delta(A)))$.

Lemma 2.2. *[11] For the $e^*\theta$-closure of a subset A of a topological space X, the following properties are hold:*

(1) $A \subseteq e^*\text{-}cl(A) \subseteq e^*\text{-}cl_\theta(A)$.

(2) If $A \in e^*\theta O(X)$, *then* $e^*\text{-}cl_\theta(A) = e^*\text{-}cl(A)$.

(3) If $A \subseteq B$, *then* $e^*\text{-}cl_\theta(A) \subseteq e^*\text{-}cl_\theta(B)$.

(4) $e^*\text{-}cl_\theta(A) \in e^*\theta C(X)$ *and* $e^*\text{-}cl_\theta(e^*\text{-}cl_\theta(A)) = e^*\text{-}\ cl_\theta(A)$.

(5) If $A_\alpha \in e^*\theta C(X)$ *for each* $\alpha \in \Lambda$, *then* $\cap\{A_\alpha | \alpha \in \Lambda\} \in e^*\theta C(X)$.

(6) If $A_\alpha \in e^*\theta O(X)$ *for each* $\alpha \in \Lambda$, *then* $\bigcup\{A_\alpha | \alpha \in \Lambda\} \in e^*\theta O(X)$.

(7) $e^*\text{-}cl_\theta(X \setminus A) = X \setminus e^*\text{-}int_\theta(A)$.

(8) $e^*\text{-}cl_\theta(A) = \cap\{U | (A \subseteq U)(U \in e^*\theta C(X))\}$.

(9) $A \in e^*O(X)$, *then* $e^*\text{-}cl_\theta(A) \in e^*R(X)$.

(10) $A \in e^*R(X)$ *if and only if* $A \in e^*\theta O(X) \cap e^*\theta C(X)$.

Lemma 2.3. *Let A be a subset of a topological space X and $x \in X$. The point x of X is an e^*-θ-cluster point of A if and only if $U \cap A \neq \emptyset$ for all e^*-θ-open U containing x.*

Proof. Let $x \notin e^*\text{-}cl_\theta(A)$.

$$\begin{aligned} x \notin e^*\text{-}cl_\theta(A) &\Leftrightarrow (\exists U \in e^*\theta C(X))(A \subseteq U)(x \notin U) \\ &\Leftrightarrow (\exists \setminus U \in e^*\theta O(X))(\setminus U \subseteq \setminus A)(x \in \setminus U) \\ &\Leftrightarrow (\exists V := \setminus U \in e^*\theta O(X, x))(V \subseteq \setminus A) \\ &\Leftrightarrow (\exists V \in e^*\theta O(X, x))(V \cap A = \emptyset) \\ &\Leftrightarrow x \notin \{x | (\forall U \in e^*\theta O(X, x))(U \cap A = \emptyset)\}. \quad \square \end{aligned}$$

Definition 2.1. A function $f : X \to Y$ is said to be contra continuous [6] (resp. contra α-continuous [12], contra precontinuous [13], contra semicontinuous [7], contra b-continuous [17], contra β-continuous [5], contra $\beta\theta$-continuous [4], contra e^*-continuous [10]) if $f^{-1}[V]$ is closed (resp. α-closed, preclosed, semiclosed, b-closed, β-closed, β-θ-closed, e^*-closed) in X for every open set V in Y.

Definition 2.2. Let A be a subset of a space X. The intersection of all open sets in X containing A is called the kernel of A [16] and is denoted by $ker(A)$.

Lemma 2.4. *[16] The following properties hold for subsets A and B of a space X.*

(1) $x \in ker(A)$ if and only if $A \cap F \neq \emptyset$ for any $F \in C(X, x)$.

(2) $A \subseteq ker(A)$.

(3) If A is open in X, then $A = ker(A)$.

(4) If $A \subseteq B$, then $ker(A) \subseteq ker(B)$.

3 Contra $e^*\theta$-continuous functions

Definition 3.1. A function $f : X \to Y$ is said to be contra $e^*\theta$-continuous (briefly c.$e^*\theta$.c.) if $f^{-1}[V]$ is e^*-θ-closed in X for every open set V of Y.

Theorem 3.1. *For a function $f : X \to Y$, the following properties are equivalent:*

(1) f is contra $e^*\theta$-continuous;

(2) The inverse image of every closed set of Y is e^*-θ-open in X;

(3) For each point $x \in X$ and each and each $V \in C(Y, f(x))$, there exists $U \in e^*\theta O(X, x)$ such that $f[U] \subseteq V$;

(4) $f[e^*\text{-}cl_\theta(A)] \subseteq ker(f[A])$ for every subset A of X;

(5) $e^*\text{-}cl_\theta(f^{-1}[B])] \subseteq f^{-1}[ker(B)]$ for every subset B of Y.

Proof.
$(1) \Rightarrow (2)$: Let $V \in C(Y)$.

$$\left.\begin{array}{r} V \in C(Y) \Rightarrow \backslash V \in O(Y) \\ (1) \end{array}\right\} \Rightarrow \backslash f^{-1}[V] = f^{-1}[\backslash V] \in e^*\theta C(X) \Rightarrow f^{-1}[V] \in e^*\theta O(X)$$

$(2) \Rightarrow (3)$: Let $x \in X$ and $V \in C(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(V \in C(Y, f(x))) \\ (2) \end{array}\right\} \Rightarrow \left.\begin{array}{r} f^{-1}[V] \in e^*\theta O(X, x) \\ U := f^{-1}[V] \end{array}\right\} \Rightarrow (U \in e^*\theta O(X, x))(f[U] \subseteq V).$$

$(3) \Rightarrow (4)$: Let $A \subseteq X$ and $x \notin f^{-1}[ker(f[A])]$.

$$\left.\begin{array}{r} x \notin f^{-1}[ker(f[A])] \Rightarrow f(x) \notin ker(f[A]) \Rightarrow (\exists F \in C(Y, f(x)))(F \cap f[A] = \emptyset) \\ (3) \end{array}\right\} \Rightarrow$$

$$\begin{aligned} &\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq F)(F \cap f[A] = \emptyset) \\ &\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U \cap A] \subseteq f[U] \cap f[A] = \emptyset) \\ &\Rightarrow (\exists U \in e^*\theta O(X, x))(U \cap A = \emptyset) \\ &\Rightarrow x \notin e^*\text{-}cl_\theta(A). \end{aligned}$$

$(4) \Rightarrow (5)$: Let $B \subseteq Y$.

$$\left.\begin{array}{r} B \subseteq Y \Rightarrow f^{-1}[B] \subseteq X \\ (4) \end{array}\right\} \Rightarrow f[e^*\text{-}cl_\theta(f^{-1}[B])] \subseteq ker(f\left[f^{-1}[B]\right]) \subseteq ker(B) \Rightarrow$$
$$\Rightarrow e^*\text{-}cl_\theta(f^{-1}[B]) \subseteq f^{-1}[ker(B)].$$

$(5) \Rightarrow (1)$: Let $V \in O(Y)$.

$$\left.\begin{array}{r} V \in O(Y) \\ (5) \end{array}\right\} \Rightarrow e^*\text{-}cl_\theta(f^{-1}[V]) \subseteq f^{-1}[ker(V)] = f^{-1}[V] \Rightarrow f^{-1}[V] \in e^*\theta C(X). \quad \square$$

Remark 3.1. From Definitions 3.1 and 2.1, we have the following diagram. None of these implications is reversible as shown by the following example:

$$\begin{array}{ccccccccc} & & \text{c.}p\text{.c.} & & & & \text{c.}\beta\text{.c.} & \leftarrow & \text{c.}\beta\theta\text{.c.} \\ & \nearrow & & \searrow & & \nearrow & & & \\ \text{c.}\alpha\text{.c.} & \leftarrow & \text{c.c.} & & \text{c.}b\text{.c.} & & \downarrow & & \downarrow \\ & \searrow & & \nearrow & & \searrow & & & \\ & & \text{c.s.c.} & & & & \text{c.}e^*\text{.c.} & \leftarrow & \text{c.}e^*\theta\text{.c.} \end{array}$$

Notation 3.1. c.c.=contra continuity, c.α.c.=contra α-continuity, c.p.c.=contra precontinuity, c.s.c.=contra semicontinuity, c.b.c.=contra b-continuity, c.β.c.= contra β-continuity, c.e^*.c.=contra e^*-continuity, c.$\beta\theta$.c.=contra $\beta\theta$-continuity, c.$e^*\theta$.c.=contra $e^*\theta$-continuity.

Example 3.1. *Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$. It is not difficult to see that*

$$e^*\theta O(X) = e^*O(X) = 2^X \setminus \{\{d\}\} \text{ and } \beta\theta C(X) = \{\emptyset, X, \{a, c, d\}, \{b, d\}, \{a, c\}, \{c\}, \{d\}\}.$$

Define the function $f : X \to X$ by $f = \{(a, c), (b, b), (c, a), (d, b)\}$. Then f is contra $e^*\theta$-continuous but it is not contra $\beta\theta$-continuous.

Other examples can be found related articles.

Definition 3.2. A function $f : X \to Y$ is said to be:

a) $e^*\theta$-semiopen if $f[U] \in SO(Y)$ for every e^*-θ-open set U of X.

b) contra $I(e^*\theta)$-continuous if for each x in X and each $V \in C(Y, f(x))$, there exists $U \in e^*\theta O(X, x)$ such that $int(f[U]) \subseteq V$.

c) $e^*\theta$-continuous [11] if $f^{-1}[V]$ is $e^*\theta$-closed in X for every closed set V of Y.

d) e^*-continuous [9] if $f^{-1}[V]$ is e^*-closed in X for every closed set V of Y.

Theorem 3.2. *Let $f : X \to Y$ be a function. If f is contra $I(e^*\theta)$-continuous and $e^*\theta$-semiopen, then f is contra $e^*\theta$-continuous.*

Proof. Let $x \in X$ and $V \in C(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(V \in C(Y, f(x))) \\ f \text{ is contra } I(e^*\theta)\text{-continuous} \end{array}\right\} \Rightarrow \left.\begin{array}{r} (\exists U \in e^*\theta O(X, x))(int(f[U]) \subseteq V = cl(V)) \\ f \text{ is } e^*\theta\text{-semiopen} \end{array}\right\} \Rightarrow$$
$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \in SO(Y))(int(f[U]) \subseteq V = cl(V))$$
$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(int(f[U])) \subseteq V). \quad \square$$

Theorem 3.3. *Let $f : X \to Y$ be a function. If f is contra $e^*\theta$-continuous and Y is regular, then f is $e^*\theta$-continuous.*

Proof. Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r}(x \in X)(V \in O(Y, f(x))) \\ Y \text{ is regular}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\exists W \in O(Y, f(x)))(cl(W) \subseteq V) \\ f \text{ is contra } e^*\theta\text{-continuous}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(W) \subseteq V). \qquad \square$$

Theorem 3.4. *Let $\{X_\alpha | \alpha \in \Lambda\}$ be any family of topological spaces. If a function $f : X \to \Pi X_\alpha$ is a contra $e^*\theta$-continuous function, then $Pr_\alpha \circ f : X \to X_\alpha$ is contra $e^*\theta$-continuous for each $\alpha \in \Lambda$, where Pr_α is the projection of ΠX_α onto X_α.*

Proof. Let $\alpha \in \Lambda$ and $U_\alpha \in RO(X_\alpha)$.

$$\left.\begin{array}{r}\alpha \in \Lambda \Rightarrow Pr_\alpha \text{ is continuous} \\ U_\alpha \in O(X_\alpha)\end{array}\right\} \Rightarrow \left.\begin{array}{r}Pr_\alpha^{-1}[U_\alpha] \in O(\Pi X_\alpha) \\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (Pr_\alpha \circ f)^{-1}[U_\alpha] = f^{-1}[Pr_\alpha^{-1}[U_\alpha]] \in e^*\theta C(X). \qquad \square$$

Definition 3.3. A function $f : X \to Y$ is called weakly e^*-irresolute [20] (resp. strongly e^*-irresolute [20]) if $f^{-1}[A]$ is e^*-θ-open in X (resp. e^*-θ-open) for every e^*-θ-open (resp. e^*-open) set A of Y.

Theorem 3.5. *Let $f : X \to Y$ and $g : Y \to Z$ and $g \circ f : X \to Z$ functions. Then the following properties hold:*

(1) If f is contra $e^*\theta$-continuous and g is continuous, then $g \circ f$ is contra $e^*\theta$-continuous.

(2) If f is $e^*\theta$-continuous and g is contra-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.

(3) If f is contra $e^*\theta$-continuous and g is contra-continuous, then $g \circ f$ is $e^*\theta$-continuous.

(4) If f is weakly e^*-irresolute and g is contra $e^*\theta$-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.

(5) If f is strongly e^*-irresolute and g is contra e^*-continuous, then $g \circ f$ is contra $e^*\theta$-continuous.

Proof. Straightforward. $\square$

4 Some fundamental properties of contra $e^*\theta$-continuous functions

Definition 4.1. A topological space X is said to be:

a) $e^*\theta$-T_0 [3] if for any distinct pair of points x and y in X, there is an $e^*\theta$-open set U in X containing x but not y or an $e^*\theta$-open set V in X containing y but not x.

b) $e^*\theta$-T_1 [3] if for any distinct pair of points x and y in X, there is an $e^*\theta$-open set U in X containing x but not y and an $e^*\theta$-open set V in X containing y but not x.

c) $e^*\theta$-T_2 [3] (resp. e^*-T_2 [10]) if for every pair of distinct points x and y, there exist two $e^*\theta$-open (resp. e^*-open) sets U and V such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Lemma 4.1. *[3] For a topological space X, the following properties are equivalent:*

(1) (X, τ) is $e^*\theta$-T_0.

(2) (X, τ) is $e^*\theta$-T_1.

(3) (X, τ) is $e^*\theta$-T_2.

(4) (X, τ) is e^*-T_2.

(5) For every pair of distinct points $x, y \in X$, there exist $U \in e^*O(X, x)$ and $V \in e^*O(X, y)$ such that e^*-$cl(U) \cap e^*$-$cl(V) = \emptyset$.

(6) For every pair of distinct points $x, y \in X$, there exist $U \in e^*R(X, x)$ and $V \in e^*R(X, y)$ such that $U \cap V = \emptyset$.

(7) For every pair of distinct points $x, y \in X$, there exist $U \in e^*\theta O(X, x)$ and $V \in e^*\theta O(X, y)$ such that e^*-$cl_\theta(U) \cap e^*$-$cl_\theta(V) = \emptyset$.

Theorem 4.1. *A topological space X is $e^*\theta$-T_2 if and only if the singletons are e^*-θ-closed sets.*

Proof. Necessity. Let $x \in X$ and X is $e^*\theta$-T_2.

$$\left.\begin{array}{r} y \notin \{x\} \Rightarrow x \neq y \\ X \text{ is } e^*\theta\text{-}T_2 \end{array}\right\} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(\exists V_y \in e^*\theta O(X, x))(U_y \cap V_y = \emptyset)$$
$$\left.\begin{array}{r} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(x \notin U_y) \\ \mathcal{A} := \{U_y | y \notin \{x\} \Rightarrow (\exists U_y \in e^*\theta O(X, y))(x \notin U_y)\} \subseteq e^*\theta O(X) \end{array}\right\} \Rightarrow$$

$$\Rightarrow X \setminus \{x\} = \bigcup \mathcal{A} \in e^*\theta O(X) \Rightarrow \{x\} \in e^*\theta C(X).$$

Sufficiency. Suppose that $\{x\}$ is e^*-θ-closed for every $x \in X$. Let $x, y \in X$ with $x \neq y$.

$$\left.\begin{array}{r} x \neq y \Rightarrow y \in X \setminus \{x\} \\ x \in X \Rightarrow \{x\} \in e^*\theta C(X) \end{array}\right\} \Rightarrow X \setminus \{x\} \in e^*\theta O(X, y).$$

Then X is $e^*\theta$-T_0. On the other hand, the notions of $e^*\theta$-T_0 and $e^*\theta$-T_1 are equivalent from Lemma 4.1. Thus X is $e^*\theta$-T_1. □

Theorem 4.2. *If f is a contra $e^*\theta$-continuous injection of a topological space X into a Urysohn space Y, then X is $e^*\theta$-T_2.*

Proof. Let $x_1, x_2 \in X$ and $x_1 \neq x_2$.

$$\left.\begin{array}{r} x_1 \neq x_2 \\ f \text{ is injective} \end{array}\right\} \Rightarrow \left.\begin{array}{r} f(x_1) \neq f(x_2) \\ Y \text{ is Urysohn} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists U\in O(Y,y_1))(\exists V\in O(Y,y_2))(cl(U)\cap cl(V)=\emptyset)\\ f \text{ is c.}e^*\theta\text{.c. at } x_1 \text{ and } x_2\end{array}\right\}\Rightarrow$$

$$\Rightarrow (\exists A\in e^*\theta O(X,x_1))(\exists B\in e^*\theta O(X,x_2))(f[A]\cap f[B]\subseteq cl(U)\cap cl(V)=\emptyset)$$

$$\Rightarrow (\exists A\in e^*\theta O(X,x_1))(\exists B\in e^*\theta O(X,x_2))(A\cap B=\emptyset).\qquad\square$$

Definition 4.2. A topological space X is said to be:

a) Weakly Hausdorff [21] (briefly weakly-T_2) if every point of X is an intersection of regularly closed sets of X.

b) Ultra Hausdorff [22] if for each pair of distinct points x and y in X, there exist clopen sets U and V containing x and y, respectively such that $U\cap V=\emptyset$.

Theorem 4.3. *Let $f:X\to Y$ be a function. Then the following properties are hold:*

(1) If f is a contra $e^*\theta$-continuous injection and Y is T_0, then X is $e^*\theta$-T_2.

(2) If f is a contra $e^*\theta$-continuous injection and Y is Ultra Hausdorff, then X is $e^*\theta$-T_2.

Proof. (1) Let $x_1,x_2\in X$ and $x_1\neq x_2$.

$$\left.\begin{array}{r}\left.\begin{array}{r}(x_1,x_2\in X)(x_1\neq x_2)\\ f\text{ is injective}\end{array}\right\}\Rightarrow\ f(x_1)\neq f(x_2)\\ Y\text{ is }T_0\end{array}\right\}\Rightarrow$$

$$\Rightarrow [(\exists V\in O(Y,f(x_1)))(f(x_2)\in V)\vee(\exists U\in O(Y,f(x_2)))(f(x_1)\in U)]$$

$$\left.\begin{array}{r}\Rightarrow (f(x_1)\notin Y\setminus V)(Y\setminus V\in C(Y,f(x_2)))\\ f\text{ is c.}e^*\theta\text{.c.}\end{array}\right\}\Rightarrow x_1\notin f^{-1}[Y\setminus V]\in e^*\theta O(X,x_2).$$

Therefore X is $e^*\theta$-T_0 and by Theorem 4.1 X is $e^*\theta$-T_2.

(2) It is not difficult to see that this item is immediate consequence of *(1)* by Lemma 4.1. $\square$

Definition 4.3. A space X is said to be:

a) $e^*\theta$-connected if X cannot be expressed as the disjoint union of two non-empty e^*-θ-open sets.

b) $e^*\theta$-normal if for each pair of non-empty disjoint closed sets can be separated by disjoint e^*-θ-open sets.

Theorem 4.4. *If $f:X\to Y$ is a contra $e^*\theta$-continuous surjection and X is $e^*\theta$-connected, then Y is connected.*

Proof. Suppose that Y is not connected.

$$Y\text{ is not connected}\Rightarrow(\exists U_1,U_2\in O(Y)\setminus\{\emptyset\})(U_1\cap U_2=\emptyset)(U_1\cup U_2=Y)$$

$$\left.\begin{array}{r}\Rightarrow U_1,U_2\in CO(Y)\\ f\text{ is c.}e^*\theta\text{.c. surjection}\end{array}\right\}\Rightarrow$$

$$\Rightarrow (f^{-1}[U_1],f^{-1}[U_2]\in e^*\theta O(X)\setminus\{\emptyset\})(f^{-1}[U_1]\cap f^{-1}[U_2]=\emptyset)(f^{-1}[U_1]\cup f^{-1}[U_2]=X).$$

This is a contradiction to the fact that X is $e^*\theta$-connected. $\square$

Theorem 4.5. *If $f : X \to Y$ is a contra $e^*\theta$-continuous closed injection and Y is normal, then X is $e^*\theta$-normal.*

Proof. Let $F_1, F_2 \in C(X)$ and $F_1 \cap F_2 = \emptyset$.

$$\left.\begin{array}{r}(F_1, F_2 \in C(X))(F_1 \cap F_2 = \emptyset)\\ f \text{ is closed injection}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(f[F_1], f[F_2] \in C(Y))(f[F_1 \cap F_2] = f[F_1] \cap f[F_2] = \emptyset)\\ Y \text{ is normal}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists V_1, V_2 \in O(Y))(f[F_1] \subseteq V_1)(f[F_2] \subseteq V_2)(V_1 \cap V_2 = \emptyset)\\ Y \text{ is normal}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists G_1, G_2 \in O(Y))(f[F_1] \subseteq G_1 \subseteq cl(G_1) \subseteq V_1)(f[F_2] \subseteq G_2 \subseteq cl(G_2) \subseteq V_2)(V_1 \cap V_2 = \emptyset)\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\begin{array}{l}\Rightarrow (f^{-1}[cl(G_1)], f^{-1}[cl(G_2)] \in e^*\theta O(X))(F_1 \subseteq f^{-1}[cl(G_1)])(F_2 \subseteq f^{-1}[cl(G_2)])\\ (f^{-1}[cl(G_1)] \cap f^{-1}[cl(G_2)] = \emptyset). \quad \square\end{array}$$

Definition 4.4. A function $f : X \to Y$ has a contra $e^*\theta$-closed graph if for each $(x, y) \notin G(f)$, there exist $U \in e^*\theta O(X, x)$ and $V \in C(Y, y)$ such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 4.2. *The graph $G(f)$ of a function $f : X \to Y$ is contra $e^*\theta$-closed in $X \times Y$ if and only if for each $(x, y) \notin G(f)$, there exist $U \in e^*\theta O(X, x)$ and $V \in C(Y, y)$ such that $f[U] \cap V = \emptyset$.*

Proof. Straightforward. $\square$

Theorem 4.6. *If $f : X \to Y$ is contra $e^*\theta$-continuous and Y is Urysohn, then f has a contra $e^*\theta$-closed graph.*

Proof. Let $(x, y) \notin G(f)$.

$$\left.\begin{array}{r}(x, y) \notin G(f) \Rightarrow y \neq f(x)\\ Y \text{ is Urysohn}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists V \in O(Y, f(x)))(\exists W \in O(Y, y))(cl(V) \cap cl(W) = \emptyset)\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \subseteq cl(V))(cl(V) \cap cl(W) = \emptyset)$$

$$\Rightarrow (\exists U \in e^*\theta O(X, x))(f[U] \cap W \subseteq f[U] \cap cl(W) = \emptyset). \quad \square$$

Theorem 4.7. *Let $f : X \to Y$ be a function and $g : X \to X \times Y$ the graph function of f, defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $e^*\theta$-continuous, then f is contra $e^*\theta$-continuous.*

Proof. Let $V \in O(Y)$.

$$\left.\begin{array}{r}V \in O(Y) \Rightarrow X \times V \in O(X \times Y)\\ g \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow f^{-1}[V] = g^{-1}[X \times V] \in e^*\theta C(X). \quad \square$$

Theorem 4.8. *If $f : X \to Y$ has a contra $e^*\theta$-closed graph and injective, then X is $e^*\theta$-T_1.*

Proof. Let $x_1, x_2 \in X$ and $x_1 \neq x_2$.

$$\left.\begin{array}{r}\left.\begin{array}{r}(x_1, x_2 \in X)(x_1 \neq x_2)\\ f \text{ is injective}\end{array}\right\} \Rightarrow f(x_1) \neq f(x_2) \Rightarrow (x_1, f(x_2)) \notin G(f)\\ G(f) \text{ is contra } e^*\theta\text{-closed}\end{array}\right\} \Rightarrow$$

$$\begin{array}{c}\Rightarrow (\exists U \in e^*\theta O(X, x_1))(\exists V \in O(Y, f(x_2)))(f[U] \cap V = \emptyset)\\ \Rightarrow (\exists U \in e^*\theta O(X, x_1))(\exists V \in O(Y, f(x_2)))(U \cap f^{-1}[V] = \emptyset)\\ \Rightarrow (\exists U \in e^*\theta O(X, x_1))(x_2 \notin U)\end{array}$$

Then X is $e^*\theta$-T_0. On the other hand, the notions of $e^*\theta$-T_0 and $e^*\theta$-T_1 are equivalent from Lemma 4.1. Thus X is $e^*\theta$-T_1. $\square$

Definition 4.5. A topological space X is said to be:

a) Strongly S-closed [6] if every closed cover of X has a finite subcover.

b) Strongly $e^*\theta C$-compact [3] if every e^*-θ-closed cover of X has a finite subcover.

c) $e^*\theta$-compact if every e^*-θ-open cover of X has a finite subcover.

d) $e^*\theta$-space if every e^*-θ-closed set is closed.

Theorem 4.9. *If $f : X \to Y$ has a contra $e^*\theta$-closed graph and X is an $e^*\theta$-space, then $f^{-1}[K]$ is closed in X for every strongly S-closed subset K of Y.*

Proof. Let K is strongly S-closed in Y and let $x \notin f^{-1}[K]$.

$$\left.\begin{array}{r}x \notin f^{-1}[K] \Rightarrow f(x) \notin K \Rightarrow (\forall y \in K)(y \neq f(x)) \Rightarrow (x, y) \notin G(f)\\ G(f) \text{ is } e^*\theta\text{-closed}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists U_y \in e^*\theta O(X, x))(\exists V_y \in C(Y, y))(f[U_y] \cap V_y = \emptyset)\\ \mathcal{A} := \{K \cap V_y | y \in K\}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\Rightarrow (\mathcal{A} \subseteq C(Y))(K = \bigcup \mathcal{A})\\ K \text{ is strongly } S\text{-closed in } Y\end{array}\right\} \Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(K \subseteq \bigcup \mathcal{A}^*)\\ U := \cap\{U_y | U_y \in \mathcal{A}^*\}\end{array}\right\} \overset{X \text{ is } e^*\theta\text{-space}}{\Rightarrow}$$

$$\Rightarrow (U \in O(X, x))(f[U] \cap K = \emptyset) \Rightarrow (U \in O(X, x))(U \cap f^{-1}[K] = \emptyset) \Rightarrow$$

$$\Rightarrow (U \in O(X, x))(U \subseteq \setminus f^{-1}[K]) \Rightarrow x \in int(X \setminus f^{-1}[K]) \Rightarrow x \in X \setminus cl(f^{-1}[K]) \Rightarrow x \notin cl(f^{-1}[K]).$$

$\square$

Theorem 4.10. *If $f : X \to Y$ is a contra $e^*\theta$-continuous surjection and X is strongly $e^*\theta C$-compact, then Y is compact.*

Proof. Let $\mathcal{B} \subseteq O(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}\left.\begin{array}{r}(\mathcal{B} \subseteq O(Y))(Y = \bigcup \mathcal{B})\\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow (\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta C(X))(X = \bigcup \mathcal{A})\\ X \text{ is strongly } e^*\theta C\text{-compact}\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow (\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*)\\ f \text{ is surjective}\end{array}\right\} \Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*).$$

$\square$

Theorem 4.11. *Let $f : X \to Y$ be a function. Then the following properties are hold:*

(1) If f is a contra $e^*\theta$-continuous surjection and X is $e^*\theta$-compact, then Y is strongly S-closed.

(2) If f is a contra $e^*\theta$-continuous surjection and X is $e^*\theta$-compact and $e^*\theta$-space, then Y is strongly $e^*\theta C$-compact.

Proof. *(1)* Let $\mathcal{B} \subseteq C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq C(Y))(Y = \bigcup \mathcal{B}) \\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta O(X))(X = \bigcup \mathcal{A}) \\ X \text{ is } e^*\theta\text{-compact}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*) \\ f \text{ is surjective}\end{array}\right\}$$

$$\Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*)$$

(2) Let $\mathcal{B} \subseteq e^*\theta C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq e^*\theta C(Y))(Y = \bigcup \mathcal{B}) \\ X \text{ is } e^*\theta\text{-space}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{B} \subseteq C(Y))(\bigcup \mathcal{B} = Y) \\ f \text{ is c.}e^*\theta\text{.c.}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*) \\ f \text{ is surjective}\end{array}\right\} \Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*).$$

□

Theorem 4.12. *If $f : X \to Y$ is a weakly e^*-irresolute surjection and X is strongly $e^*\theta C$-compact, then Y is strongly $e^*\theta C$-compact.*

Proof. Let $\mathcal{B} \subseteq e^*\theta C(Y)$ and $Y = \bigcup \mathcal{B}$.

$$\left.\begin{array}{r}(\mathcal{B} \subseteq e^*\theta C(Y))(Y = \bigcup \mathcal{B}) \\ f \text{ is weakly } e^*\text{-irresolute}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\mathcal{A} := \{f^{-1}[B] | B \in \mathcal{B}\} \subseteq e^*\theta C(X))(X = \bigcup \mathcal{A}) \\ X \text{ is strongly } e^*\theta C\text{-compact}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{r}(\exists \mathcal{A}^* \subseteq \mathcal{A})(|\mathcal{A}^*| < \aleph_0)(X = \bigcup \mathcal{A}^*) \\ f \text{ is surjective}\end{array}\right\} \Rightarrow$$

$$\Rightarrow (\mathcal{B}^* := \{f[A] | A \in \mathcal{A}^*\} \subseteq \mathcal{B})(|\mathcal{B}^*| < \aleph_0)(Y = \bigcup \mathcal{B}^*). \qquad \square$$

We recall that the product space $X = X_1 \times \ldots \times X_n$ has property $P_{e^*\theta}$ [3] if A_i is an $e^*\theta$-open set in a topological space X_i for $i = 1, 2, \ldots n$, then $A_1 \times \ldots \times A_n$ is also $e^*\theta$-open in the product space $X = X_1 \times \ldots \times X_n$.

Theorem 4.13. *Let $f : X_1 \to Y$ and $g : X_2 \to Y$ be two functions, where*

(i) $X = X_1 \times X_2$ has the property $P_{e^*\theta}$,

(ii) Y is a Urysohn space,

(iii) f and g are contra $e^*\theta$-continuous,

then $\{(x_1,x_2)|f(x_1)=g(x_2)\}$ *is* $e^*\theta$*-closed in the product space* $X=X_1\times X_2$*.*

Proof. Let $(x_1,x_2)\notin A:=\{(x_1,x_2)|f(x_1)=g(x_2)\}$.

$$\left.\begin{array}{r}(x_1,x_2)\notin A\Rightarrow f(x_1)\neq g(x_2)\\ Y \text{ is Urysohn}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{r}(\exists V_1\in O(Y,f(x_1)))(\exists V_2\in O(Y,g(x_2)))(cl(V_1)\cap cl(V_2)=\emptyset)(cl(V_1),cl(V_2)\in RC(Y))\\ f \text{ and } g \text{ are c.}e^*\theta\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{r}(f^{-1}[cl(V_1)]\in e^*\theta O(X_1,x_1))(g^{-1}[cl(V_2)]\in e^*\theta O(X_2,x_2))\\ X=X_1\times X_2 \text{ has the Property } P_{e^*\theta}\end{array}\right\}\Rightarrow$$

$$\Rightarrow((x_1,x_2)\in f^{-1}[cl(V_1)]\times g^{-1}[cl(V_2)]\in e^*\theta O(X_1\times X_2))(f^{-1}[cl(V_1)]\times g^{-1}[cl(V_2)]\subseteq\setminus A)\Rightarrow$$

$$\Rightarrow\setminus A\in e^*\theta O(X_1\times X_2)\Rightarrow A\in e^*\theta C(X_1\times X_2).$$

□

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