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Double Down-Conversion and induced coherence

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Abstract

The behaviour of the entangled two-particle system arising from down-conversion is analyzed semiclassically in terms of the induced coherence and the conditional state. The result is tested with the help of two fundamental interferometry experiments involving the frustrated pair creation, one of them with quantum markers and with erasers insertable in a delayed-choice mode. The capability of the conditional pilot wave of influencing the particle emission is discussed.

Key work: Double-down conversion; induced coherence.

Conversión invertida doble y coherencia inducida

Resumen

La conducta de un sistema atrapado de dos partículas en la conversión invertida se analiza semiclasicamente en función de la coherencia inducida y el estado condicional. Los resultados se prueban con la ayuda de dos experimentos fundamentales de interferometria, los cuales se relacionan con la creación frustrada de dos pares, uno de ellos con marcadores cuánticos y con borradores insertados en el método de decaimiento escogido. La capacidad de la onda piloto condicional para influenciar la emisión de la partícula se discute.

Palabras clave: Conversión invertida doble; coherencia inducida.

1. Introduction

The extended Feynman rule, that is the path distinguishability rule or which-path information, is one of the most fundamental quantum features only comparable with the uncertainty principle. However, this rule which never fails is used in the guise of a magic tool, no attempt whatsoever being made so as to understand its physical origin. In fact, it is claimed that such an innermost quantum behaviour cannot be given any classical-like explanation. To our mind, a change of attitude is necessary. Accordingly, by having recourse to the concept of the conditional state (1), introduced so as to account for the Bell's inequality violation, we have shown that the fundamental types of complementarities and which-path information can be justified semiclassically (2), and on the grounds of various key experiments we have verified the fundamental importance of the conditional state (3). All these experiments and in particular those involving the induced coherence (4–7) bring support to the idea of a nonlocal but real wave-like mechanism underlying the conditional state, that is the conditional pilot wave (1).

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In the present note, by means of a semiclassical analysis similar to that of the doubleslit presented in (2), we shall be able to show the crucial role of the conditional state in justifying the unusual faculty of the induced coherence for breaking the complementarity of the coincidence and the one-particle fringes. We shall use the quantum formulation to demonstrate that the conditional state is a necessary ingredient for the obtention of the corresponding entangled coincidence probability. Then, by having recourse to two fundamental experiments which involve the frustrated pair creation and the quantum eraser (6, 7), we shall verify that all types of coincidence and one-particle patterns, including those arising in a delayed-choice mode, can in fact be accounted for in terms of the due presence or absence of the conditional state. This will allow us to discuss the influence of the boundary upon the particle emission.

2. Induced coherence

The relative phase of the pair of photons created by down-conversion is not determined by the matching condition (8-10), whereas their total phase depends crucially upon the exact place where this process occurs. Hence, down-conversion is normally an incoherent process in the sense that the successive events occurring with the same preparation are in general not coherent with each other. This impedes the apparition of the once-particle fringes in accordance with their complementarity with respect to the coincidence fringes (2). However, the necessary coherence for both types of fringes can be obtained under the influence of an inducing wave whose path and phase control the locus of the down-conversion. In fact, considering semiclassically a combination of two waves each of which can play the role of the inducing wave, and imposing a natural constraint upon the down-conversion point, we are going to show that the coherence required for obtaining simultaneously two-particle and one-particle oscillations arises from the phase continuity at the locus and moment of the down-conversion.

In Figure 1 the impinging wave *c´* is down-converted into *a´* and *b´* at the point P_1. The potential inducing waves *a* and *b* go through the same point P_1 and superposes themselves to *a´* and *b´* respectively. We consider the possibility that the same process could occur at another point P_2. The wave length of the impinging wave is λ_0, whereas λ is the wave length of the "and of the emitted waves. Clearly,

$$\delta\phi_a \equiv \delta\phi_{a'} \equiv -\frac{2\pi}{\lambda}\Delta_a \qquad [1a]$$

$$\delta\phi_b \equiv \delta\phi_{b'} \equiv -\frac{2\pi}{\lambda}\Delta_b \qquad [1b]$$

$$\delta\phi_c \equiv -\frac{2\pi}{\lambda_0}\Delta_{c'} \qquad [1c]$$

are the phase variations corresponding to the path differences
$\Delta_a := d\cos(\alpha-\beta), \Delta_b := -d\cos(\alpha-\beta)$ *and*
$\Delta_{c'} := d\cos(\beta)$, which arise when the locus of the down-conversion changes from P_1 to P_2. Then, taking into account the continuity of the phases of the impinging and the outgoing waves, we require

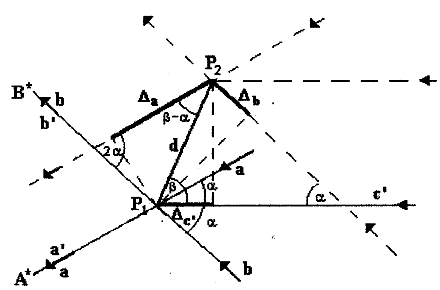

Figure 1. At the down-conversion point P_1, the incident beam c' is down-converted into a' and b', which superposes to a and b respectively each of which can act as an inducing wave. The same can occur at another down-conversion point P_2.

$$\Delta\phi_{a'} + \Delta\phi_{b'} = \delta\phi_{c'} \quad [2a]$$

where, similarly to

$$\Delta\phi_{a} := \delta\phi_{a} \quad [2b]$$

$$\Delta\phi_{b} := \delta\phi_{b} \quad [2c]$$

$$\Delta\phi_{c'} := \delta\phi_{c'} \quad [2d]$$

$\Delta\phi_{a'}$ and $\Delta\phi_{b'}$ denote the phase variations accumulated at $P = P_2$ with respect to the phases corresponding to the analogous paths ending at P_1. Indeed, relation [2a] is a consequence of the quantum-like phase continuity condition

$$\phi_{a'}(P,T) + \phi_{b'}(P,T) = \phi_{c'}(P,T) \quad [3a]$$

implied by the simultaneity and the energy conservation of the down-conversion process (8-10) at $T := t_{c'}(P) = t_{a'}(P) = t_{b'}(P)$ and $P := P_1, P_2$ Hence, we use [1ab,2] so as to get

$$\Delta\phi_{a'} - \Delta\phi_{a} + \Delta\phi_{b'} - \Delta\phi_{b} = \delta\phi_{c'} - \delta\phi_{a} - \delta\phi_{b} \quad [3b]$$

which corresponds to an arbitrary change of the down-conversion point from P_1 to P_2. Here we introduce the external phase shifts $\Delta_{ext}\phi_a, \Delta_{ext}\phi_b$ and $\Delta_{ext}\phi_{c'}$ obtaining

$$\Delta_{tot}\phi_{a} := \Delta\phi_{a} + \Delta_{ext}\phi_{a} \quad [4a]$$

$$\Delta_{tot}\phi_{b} := \Delta\phi_{b} + \Delta_{ext}\phi_{b} \quad [4b]$$

$$\Delta_{tot}\phi_{c'} := \Delta\phi_{c'} + \Delta_{ext}\phi_{c'} \quad [4c]$$

$$\Delta_{tot}\phi_{a'} + \Delta_{tot}\phi_{b'} := \Delta\phi_{a'} + \Delta\phi_{b'} + \Delta_{ext}\phi_{c'} \quad [4d]$$

We now suppose that at least one of the ingoing waves *a* or *b* exhibits the same polarization as the impinging wave *c´*, so as to serve as inducing wave. Under this condition, and assuming that *a* and *b* and *c´* are originated from a common source and therefore are correlated, so as to grant that they intersect exactly at the down conversion point a constraint upon the possible displacements of this point arises, which analogously to [2a, 3a] can be expressed by

$$\delta\phi_{a} + \delta\phi_{b} := \delta\phi_{c'} \quad [5a]$$

Thus, the down conversion point only fluctuates on the plane perpendicular to c', *and relation [3b] becomes*

$$\Delta_{tot}\phi_{a'} - \Delta_{tot}\phi_{a} + \Delta_{tot}\phi_{b'} - \Delta_{tot}\phi_{b} = \Delta_{ext}\phi_{c'} - \Delta_{ext}\phi_{a} - \Delta_{ext}\phi_{b} \quad [5b]$$

This shows that the coincidence pattern and the one particle patterns are both controlled by any of the three external shifts with the same weight. In fact, relation [5b], deduced from the phase continuity [3a] under the constraint [5a], constitutes the coernce condition which determines these patterns. We underline that this condition only needs one inducing wave. Accordingly, by double induced coherence we mean that any of the ingoing waves a and b can alternatively act as the inducing wave.

The important result of the above semiclassical analysis consists in the coherence condition [5b]. From a purely quantum mechanical viewpoint, we say that the states $|a\rangle$ and $|a'\rangle$ as well as $|b\rangle$ and $|b'\rangle$ are distinguishable and then incoherent, but become indistinguishable and coherent under the effect of the induced coherence. Formally, regarding the states $|a\rangle$ and $|b\rangle$ as potentially distinguishable from $|a'\rangle$ and $|b'\rangle$ respectively, he corresponding entangled two-particle quantum state may be written as

$$|\psi\rangle := \frac{1}{\sqrt{2}}\left[\exp i(\phi_a - \phi_b)|ab\rangle + \exp i\phi_{c'}|a'b'\rangle\right] \quad [6a]$$

where $\phi_{c'} = \phi_{a'} + \phi_{b'}$ in accordance with [3a]. The corresponding quantum probability for the coincidence rate at $A^* B^*$ reads

$$I_{A^*B^*} \equiv \frac{1}{4}[1 + \cos(\phi_a + \phi_b - \phi_{c'})] \quad [6b]$$

for which the presence of the conditional state in one of the two channels, that is either $\phi_a - \phi_{a'} = 0$ or $\phi_b - \phi_{b'} = 0$, is being assumed. Consistently, by letting the two pairs of states be indistinguishable; that is to say, by dropping the prime in the ket $|a' b'\rangle$, the expression [6a] properly renormalized becomes

$$|\psi\rangle \equiv \frac{1}{2}\left[\exp i(\phi_a - \phi_b) + \exp i\phi_{c'}\right]|ab\rangle \qquad [6c]$$

which is a direct product state with a correlated overall phase. This correlation is trivial; inasmuch as it does not make an entangle state. However the quantum prediction of the count rate at A* and B* separately derived from [6c] reads

$$I_{A^*} \equiv I_{B^*} \equiv \frac{1}{2}[1 + \cos(\phi_a + \phi_b - \phi_{c'})] \qquad [6d]$$

which is identical in form with [6b]. We see that the coincidence and the one-particle oscillations vary in the same way in accordance with the coherence condition [5b] implied semiclassically by [3a, 5a]. This condition grants the necessary coherence for the one-particle fringes leaving room for the conditional state underlying the coincidence fringes. Accordingly, the conditional state needs to be used for obtaining the joint probability [6b].

3. Experimental test

One of the most interesting interferometry setup is that realized in a first stage by Herzog, Rarita, Weinfurter and Zeilinger (6), then enlarged by Herzog, Kwiat, Weinfurter and Zeilinger (7). These two experiments show all the crucial relationships among the which-path informations and both the one-particle and the two-particle patterns. The first one has recourse to the indistinguishability created by the mechanism of induced coherence so as to break the intrinsic incompatibility between the one-particle and the coincidence fringes. The second experiment introduces an additional which-path information in the form of quantum markers capable of destroying all kind of patterns. Then, by means of quantum erasers one of which can be used in a delayed mode, the which-path information which destroys the coincidence fringes is removed; but not the one which by eliminating the induced coherence destroys the one-particle fringes. In this way, the mutual exclusivity of the two kinds of fringes comes back with all its strength.

The experimental arrangement, based on the interferometer introduced in (6), is supplemented in (7) by means of rotators and linear polarizers as depicted in Figure 2. A vertically polarized photon c emitted by a source S is either down-converted at D into a and b, or denoted by c´ goes ahead up to the mirror C and comes back to D, where it is down-converted into a´ and b´. Thus, a´ and *b´* superpose themselves to a and b respectively, which have been sent back through *D* by the respective mirrors *A* and *B*. The rotators R_A and R_B, if present, change the vertical polarization of the reflected ingoing photons *a* and *b* into the horizontal polarization; whereas *a´* and *b´* remain vertically polarized. In order to recover a common linear polarization, the linear polarizers P_{A*} and P_{B*}, which in behalf of simplicity we supposed either parallel or perpendicular to each other at 45° from the vertical, can be introduced either permanently or, one of them, in a delayed mode.

Let us consider the first experiments in which none of the rotators and polarizers are present. Clearly, *a*, *b* and *c* are coherent and so are *a´*,*b´* and *c´*. In contrast, a and a' as well as *b* and *b´* are potentially incoherent; because in principle the transversal position of the down-conversion point is not well determined. This potential incoherence, and then distinguishability, turns itself into coherence and indistinguishability inasmuch as one of the ingoing waves *a* and *b*, reflected by the corresponding mirror, acting as a coherence inducing wave yields a conditional state. In fact, the phases of these waves are correlated to those of the outgoing waves in

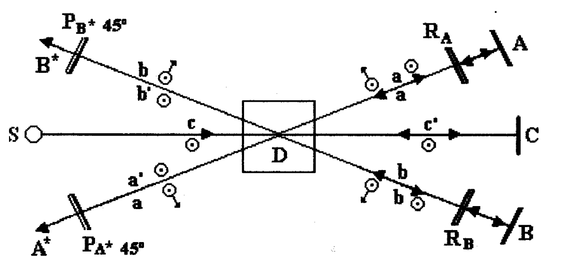

Figure 2. The vertically polarized incident photon c is either down-converted at D into a and b, or proceeds along c' and is reflected by the mirror C so as to be down-converted at D into a' and b', which superpose to a and b produced by the first down-conversion and reflected by the mirrors A and B. The rotator Ra and RB change the polarization of a and b from vertical to horizontal and the linear polarizers PA and PS at 45º from the vertical, insertable in a delayed mode, recover a common polarization. the down-conversion point within D is potentially free, such as P_1 or P_2 of Figure 1. This makes possible the arising of the conditional state under the induced coherence provided by either a or b with the corresponding rotator and polarizer removed, and therefore yields the simultaneous presence of the coincidence and the one-particle patterns.

the way described by the coherence condition [5b]. This condition allows that that either *aa´* or *bb´* superpose themselves constructively so as to constitute the conditional state, no matter the shift of any of the mirror positions. Hence, under this shift, *bb´* and *aa´* exhibit coincidence and one-particle oscillations simultaneously. Clearly, the visibility v_2 of the two-particle coincidence fringes and the visibilities $v_{aa'}$ and $v_{bb'}$ of the two one-particle fringes are such that

$$v_{aa'} \sim v_{bb'} < v_2 < 1 \qquad [7]$$

as may be inferred from [6bd]. The inequalities in this relation accounts for a non interfering background count rate; whereas a difference between $v_{aa'}$ and $v_{bb'}$ can arise from an asymmetry of the down-conversion process. All these theoretical predictions are confirmed by the observational data provided in (6). We emphasize that in this experiment the complementarity of the one-particle and the coincidence fringes is broken by the induced coherence. On the other hand, the presence of coincidence fringes in spite of a single two-channel output, is a striking evidence of the frustrated emission caused by the boundaries via interference.

In the second experiment, also depicted in Figure 2, the beams *a* and *b* are still coherent and so are *a´* and *b´*. However, with both R_A and R_B present, *a* and *a´* as well as *b* and *b´* are incoherent and therefore distinguishable. Consistently, by measuring the polarization for example at A^*, one can distinguish between a and a', and then between *b* and *b´* without measuring at B^*. Clearly, because of the lack of induced coherence, the conditional state and therefore the one-particle fringes are forbidden in presence of the two rotators in spite of the two polarizers. In contrast, as shown in [Figure 3ac], reduced coincidence fringes can be observed at A^*B^*, provided both polarizers P_{A*} and P_{B*} be present, in either the permanent or the delayed mode; but these trivial fringes are not a product of entanglement and therefore cannot violate Bell's inequality. On the other hand, as shown in [Figure 2ac], nontrivial coincidence fringes can be observed using only one of the rotators while the one-particle fringes are observable in the channel which contains such a rotator and the corresponding polarizer. This does not contradict the intrinsic complementarity between the one-particle and the coincidence patterns, which is removed because the second down-conversion is induced by the firstly down-converted wave reflected without rotation. We emphasize that the wave endowed with the coherence among distinct events needed for the one-particle fringes, is not the one which serves as inducing wave, which is the one capable of originating the conditional state in each event. All these features are actually observed in the experiment (7) in quantitative agreement with the visibility relation.

It is important to realize that the conditional state cannot arise if in the region of the down-conversion the two superposed waves are linearly polarized in orthogonal directions, what is a consequence of the lack of induced coherence. In fact, in Figure 2 with both rotators present, the coincident detections does not show the characteristic pattern capable of violating Bell's inequality, even though the detectors are permanently preceded by the respective diagonal polarizers. In contrast, the diagonal polarizers can be used to pull the already present probability information out of the correlated non-conditional pilot waves, what constitutes a case of trivial delayed-choice. These polarizers are merely analyzers forming part of the detection mechanisms, so they can be inserted in an extreme delayed mode just before the detection of the corresponding particle, even after the detection of the other particle. Accordingly, in presence of rotators, the coincidence fringes are to be justified by means of absorption at the polarizers, rather than by frustrated emission at the down-converter.

4. Conclusion

The extraordinary results presented in (6, 7), have been given a physical justification not attempted in those papers which only relays on the quantum algorithm of the path distinguishability. The coherence condition leaves open the possibility that, on the basis of the conditionality hypothesis, the superposition in one of the two ending channels be entirely constructive so as to yield a conditional quantum state in agreement with the principle of stationarity of the wave function (1). As different from the two double-slits (2), the induced coherence implies the appearance of the conditional state in one of the two channels together with the one-particle oscillations in the other, such a conditional state being not only permitted, but needed for the one-particle fringes. The essential role of the conditional state has been stressed and the globality of the pilot wave has been emphasized; but at the same time, the c apability of this wave for controlling the particle emission has been shown to be not unrestricted.

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