CIENCIA 9(1), 46-49, 2001
Maracaibo, Venezuela

Gauge invariance for the massive axion*

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Recibido: 09-06-1999 Aceptado: 21-02-2001

Abstract

A massive gauge invariant formulation for scalar (ϕ) and antisymmetric (C_{mnp}) fields with a topological coupling, which provides a mass for the axion field, is considered. The dual and local equivalence with the nongauge invariant proposal is established, but on manifolds with non-trivial topological structure both formulations are not globally equivalent.

Key words: Axion field; Gauge invariance; topological coupling.

Invariancia de calibre para el axión masivo

Resumen

Se considera una formulación invariante de calibre que acopla a través de un término topológico un campo escalar (ϕ) con un campo antisimétrico (C_{mnp}). Se establece la equivalencia dual y local con la propuesta no invariante de calibre, pero ambas formulaciones no son globalmente equivalentes.

Palabras clave: Acoplamiento topológico; campo de axión; invariancia de calibre.

1. Introduction

In four dimensions a massless (pseudo)scalar field: the axion, is dual to the antisymmetric field B_{mn} (only if derivative couplings are considered, therefore massive terms are excluded) as a particular case of the general duality between p and $D-p-2$ forms in D dimensions. Since non-perturbative effects break the local Peccei-Quinn symmetry for the axion field, in order to give mass to the axion, the duality between a massive axion and an antisymmetric field was considered an enigma until two independent approaches (1, 2) were developed recently. Now, is understood that take into account non-perturbative effects, the usual duality between a massless scalar field and an antisymmetric field B_{mn} is not broken, but replaced by the duality between a massive scalar ϕ and an massive antisymmetric field C_{mnp}. An early attempt to undestand this duality was considered in the reference (3). A characteristic feature of this duality is the lost of abelian gauge invariance for the antisymmetric field. In this article, we will show that a gauge invariant theory, which involved a topological cou-

* Trabajo presentado en el Primer Congreso Venezolano de Física, Facultad de Ciencias, Universidad de Los Andes. Mérida, del 7 al 12 de diciembre de 1997.

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pling and considered several years ago (4) in the context of the U(1) problem, is locally equivalent to the non gauge invariant proposal. This equivalence is similar to what happen in three dimensions for massive topologically and self-dual theories (5) and the Proca and massive topologically gauge invariant theories in four dimensions (6). We will study the equivalence through the existence of a master action from which local and global considerations are established.

2. The Gauge Invariant Model

An illustrative model for the massive axion is given by the following master action (7)

$$I =< \frac{1}{2} \upsilon_m \upsilon^m + \phi\, \partial_m \upsilon^m - \frac{1}{2} m^2 \phi^2 >, \qquad [1]$$

where υ_m is a vector field and ϕ is a scalar field (< > denotes integration in four dimensions). Eliminating the field υ_m through its equation of motion: $\upsilon_m = \partial_m \phi$, the action for a massive scalar is obtained, while using the equation of motion, obtained by varying the scalar field $\phi \left(\phi = \frac{1}{m^2} \partial_m \upsilon^m \right)$ we have

$$I_\upsilon = \frac{1}{2} < \upsilon_m \upsilon^m + \frac{1}{m^2} (\partial_m \upsilon^m)^2 > \qquad [2]$$

and the propagator corresponding to the field υ_m is $\eta_{mn} - \frac{k_m k_n}{k^2 + m^2}$, which is just equal to those discussed in (1). A simple way to show the duality, rely on introducing the dual of the vector field $\upsilon^m = \frac{1}{3!} m \varepsilon^{mnpq} C_{npq}$ in the action I_υ, yielding the master action

$$I_{M1} =< -\frac{1}{2 3!} m^2 C^{mnp} C_{mnp} + \frac{1}{3!} m \varepsilon^{mnpq} \phi \partial_m C_{npq} - \frac{1}{2} m^2 \phi^2 > \qquad [3]$$

from which the duality is easily infered. In fact, eliminating the scalar field C_{mnp} (or ϕ) through its equation of motion, the action for a massive scalar field (or the massive antisymmetric field C_{mnp}) is obtained. In anycase, the gauge invariance is spoiled. Now, we can ask whether there really exist an invariant gauge theory compatible with a massive term for the axion field. The answer is possitive. We will show that the following action:

$$I_{M2} =< -\frac{1}{2} \partial_m \phi \partial^m \phi - \frac{1}{24!} G_{mnpq} G^{mnpq} - \frac{m}{6} \varepsilon^{mnpq} C_{mnp} \partial_q \phi >, \qquad [4]$$

where

$$G_{mnpq} \equiv \partial_m C_{npq} - \partial_n C_{mpq} + \partial_p C_{mnq} - \partial_q C_{mnp}$$

is the field strenght associated to the antisymmetric field C_{mnp}, is locally equivalent to I_{M1}, describing the propagation of a massive scalar excitation: a massive axion. Note that the coupling term is an extensión of the usual *BF* term and the action is invariant under the abelian gauge transformations

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi \phi = 0 \qquad [5]$$

This action was considered previously in ref. (4), as a generalization to four dimensions of the Schwinger model in two dimensions.

Let us see, how this action is related to the propagation of a massive axion and why the equivalence with the non gauge invariant action must hold. Rewritten down the action (equation [4]) by introducing $F^{mnp} \equiv \varepsilon^{mnpq} F_q$ as the dual tensor of F_m, we can eliminate F^{mnp} through its equation of motion: $F^{mnp} = -mC^{mnp}$ and substituing, the action for the massive antisymmetric field C^{mnp} appears. Going on an additional step, the dual of the antisymmetric field $C^{mnp} = \frac{1}{m} \varepsilon^{mnpq} \upsilon_q$ is introduced, and the action for the vector field υ_m, equation [2], is obtained. On the other hand, if we introduce $\lambda \equiv -\frac{1}{4}\varepsilon^{mnpq}G_{mnpq}$ as the dual of the strenght field G_{mnpq} into the action (4), we observe that λ plays the role of an auxiliary field, whose elimination through its equation of motion ($\lambda = -m\phi$) lead to the action of a massive scalar field.

It is worth recalling, since the action is expressed only in derivatives of the scalar field, that the dual theory can be achieved, reemplacing $\partial_m\phi$ by $\frac{1}{2}l_m$ and add a BF term: $\frac{1}{4}l_m\varepsilon^{mnpq}\partial_n B_{pq}$ (8). The dual action is (4):

$$I_d = <-\frac{1}{24!}G_{mnpq}G^{mnpq} - \frac{1}{23!}(mC_{mnp} - H_{mnp})(mC^{mnp} - H^{mnp})>, \qquad [6]$$

where $H_{mnp} = \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength of the antisymmetric field B_{mn}, which was introduced in the BF term. This action just describes the interaction of open menbranes whose boundaries are closed strings (9) and is invariant under the following gauge transformations

$$\delta C_{mnp} = \partial_m\xi_{np} + \partial_n\xi_{pm} + \partial_p\xi_{mn},$$
$$\delta B_{mn} = \partial_m\lambda_n - \partial_n\lambda_m - m\xi_{mn}. \qquad [7]$$

The ξ gauge transformation allows us gauged away the antisymmetric field B_{mn}, leading to the massive antisymmetric field C_{mnp} action.

3. The Equivalence

Now, we are going on to show the equivalence. Let us take the following master action

$$I_M = <-\frac{1}{23!}m^2 a_{mnp}a^{mnp} - \frac{1}{2!}m^2\psi^2 + \frac{1}{4!}m\varepsilon^{mnpq}\psi G_{mnpq} + \frac{1}{3!}m\varepsilon^{mnpq}(a_{mnp} - C_{mnp})\partial_q\phi > \qquad [8]$$

Independent variations in a_{mnp}, ψ, C_{mnp} and ϕ lead to the following equations of motion

$$a^{mnp} = \frac{1}{m}\varepsilon^{mnpq}\partial_q\phi, \qquad [9]$$

$$\psi = \frac{1}{4!m}\varepsilon^{mnpq}G_{mnpq}, \qquad [10]$$

$$\varepsilon^{mnpq}\partial_m(\psi - \phi) = 0, \qquad [11]$$

and

$$\varepsilon^{mnpq}\partial_q(a_{mnp} - C_{mnp}) = 0. \qquad [12]$$

Replacing the expressions for a_{mnp} and ψ given by equations [9] and [10] into I_M, the gauge invariant action I_{M2} obtained. On the other hand, the solutions of the equations of motion (11) and (12) are:

$$\phi - \psi = \omega, \qquad C_{mnp} - a_{mnp} = \Omega_{mnp}, \qquad [13]$$

where ω and Ω_{mnp} are 0 and 3-closed forms, respectively. Locally, we can set

$$\omega = constant,$$
$$\Omega_{mnp} \equiv L_{mnp} = \partial_m l_{np} + \partial_n l_{pm} + \partial_p l_{mn}, \qquad [14]$$

and subtituing into I_M, we obtain the following "Stuckelberg" action

$$I_s = <-\frac{1}{23!}m^2(C_{mnp} - L_{mnp})(C^{mnp} - L^{mnp}) - \frac{1}{2}m^2(\phi - \omega)^2 + \frac{1}{4!}m\varepsilon^{mnpq}(\phi - \omega)G_{mnpq} > \qquad [15]$$

This action is invariant under

$$\delta_\xi C_{mnp} = \partial_m\xi_{np} + \partial_n\xi_{pm} + \partial_p\xi_{mn},$$
$$\delta_\xi l_{mn} = \xi_{mn} \qquad [16]$$

which allow us gauged away the l_{mn} field and recover I_{M1} (we have redefined $\phi - \omega$ as ϕ since ω is a constant). In this way, the local equivalence is stated. This local equivalence can also be established from a hamiltonian point of view and will be reported elsewhere (10). On the other hand, we can consider $\psi = \phi - \omega$ and $a_{mnp} = C_{mnp} - \Omega_{mnp}$ as the general solutions in order to obtain the following gauge invariant action:

$$\bar{I}_M =< -\frac{1}{2\,3!}m^2(C_{mnp}-\Omega_{mnp})(C^{mnp}-\Omega^{mnp}) - \frac{1}{2!}m^2(\phi-\omega)^2 + \frac{1}{4!}m\varepsilon^{mnpq}(\phi-\omega)G_{mnpq} - \frac{1}{3!}m\varepsilon^{mnpq}\Omega_{mnpq}\partial_q\phi > \quad [17]$$

This action is global and locally equivalent to I_{M2}, where the topological sectors not present in I_{M1} are considered. Indeed, the equations of motion which are obtained after performing independent variations on C_{mnp}, Ω_{mnp}, ϕ and ω in $\bar{I}_M$ are:

$$m(C^{mnp}-\Omega^{mnp})-\varepsilon^{mnpq}\partial_q(\phi-\omega)=0$$
$$m(C^{mnp}-\Omega^{mnp})-\varepsilon^{mnpq}\partial_q\phi=0 \quad [18]$$

and

$$m(\phi-\omega)-\frac{1}{3!}\varepsilon^{mnpq}\partial_m C_{npq}+\frac{1}{3!}\varepsilon^{mnpq}\partial_m\Omega_{npq}=0$$
$$m(\phi-\omega)-\frac{1}{3!}\varepsilon^{mnpq}\partial_m C_{npq}=0, \quad [19]$$

from which is easily deduced that ω and Ω_{mnp} are 0 and 3-forms closed, respectively, i.e., $\varepsilon^{mnpq}\partial_q\omega = 0 = \varepsilon^{mnpq}\partial_m\Omega_{npq}$. Taking into account this last result and applying the differential operator $\varepsilon^{mnpq}\partial_q$ on the set first order differential equationss given by (30), the equations of motion for the gauge invariant action I_{M2} are obtained, equations [8] and [9].

Finally, we can eliminate ϕ and C_{mnp} to achieve

$$\bar{I}_{M1} = I_{M1[a,\psi]} - I_{top[\omega,\Omega]}, \quad [20]$$

where

$$I_{top[\omega,\Omega]} =< \frac{1}{3!}m\varepsilon^{mnpq}\Omega_{mnp}\partial_q\omega > \quad [21]$$

is the extension of the BF term for the topological coupling between 0 and 3-forms in four dimensions. From this result, we have that the partition funtions of I_{M1} and I_{M2} differ by a topological factor.

$$Z_{M2} = Z_{top}Z_{M1} \quad [22]$$

In general, on manifolds with non trivial topological structure $Z_{top} \neq 1$. Only when the manifold has a trivial structure, we must have $Z_{top} \equiv 1$, reflecting the local equivalence.

Sumarizing, we have seen that a gauge invariant description for massive axions is possible which is (locally) equivalent to the non-gauge invariant proposal. Several aspects of this proposal are under considerations: a detailed hamiltonian description for both proposal of generating mass for the axion and a complete BRST analysis of the gauge invariant model considered in this paper (10).

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